Singular Value Decomposition based Phi Domain Equalization for Multi-Carrier Communication System

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***Abstract*—Phi transform provides flexible way for construction of basis functions with desired parameters. This feature makes it highly eligible for utilization in multi-carrier communication systems. This paper is devoted to the problem of construction of an algorithm, which is capable to mitigate inter-block interference in Phi transform based communication system. Single equalization method and several channel estimation algorithms are proposed and their performance is compared. Simulation results of communication systems with various combinations of signal structure and channel estimators are presented and compared.**

***Index Terms*—Broadband communication, code division multiplexing, telecommunication control, communication switching.**

I. INTRODUCTION

Multicarrier (MC) communication systems have gained a noticeable interest in the academic and industrial communities during the last decades. One of the most popular MC communication technologies is orthogonal frequency division multiplexing (OFDM) [[1]]. This modulation technique utilizes sinusoidal basis functions, which are well explored and suitable to MC transmission due to low inter-carrier interference and ease of equalization. On other hand, utilization of sinusoids requires high precision digital-to analog and analog-to digital converters and power amplifiers with high dynamic range.

Broadband communication systems based on non-sinusoidal basis functions [[2]], compared to sinusoidal ones, can provide greater flexibility in terms of dynamic range, security and interference immunity. During last years our group is focused on research on new kind of parametrical unitary transforms – Phi transforms. Phi transform [[3]] is based on plane rotation in Hilbert space. Experiments by Oka [[4]] and our group [[5]] has shown that rotation provides an elegant way for producing basis functions with desired characteristics and can lead to new and efficient designs of communication systems.

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II. PROBLEM DEFINITION

In our recent publication [[6]] we have shown that equalization in domains other than time domain or frequency domain cannot be performed by the traditional techniques. Most time domain equalization techniques are based on assumption that communication channel behaves like a finite impulse response filter (FIR). On other hand frequency-domain equalizers exploit feature that, if transmitted sequences are periodic, then channel behaves like a complex scalar multiplier.

Goal of this contribution is to describe an equalizer, which is capable to equalize signals in Phi domain. Described equalizer is compared to the traditional solutions and computer simulation results are provided.

III. SYSTEM MODEL

In communication system where input signal $x(t)$ is being transferred via communication channel with time domain impulse response $h(t)$, the output signal can be described as follows

$$y(t) = h(t) * x(t). \tag{1}$$

If transmitted signal is sampled with sampling period T_s and grouped into block $\mathbf{x}$, and there is no inter-block interference (IBI), then received block $\mathbf{y}$ can be described by the following matrix equation

$$\mathbf{y} = \mathbf{Hx} + \mathbf{w}, \tag{2}$$

where $\mathbf{H}$ is channel matrix and $\mathbf{w}$ is additive noise vector. If channel is static and linear, then multipath propagation in communication channel can be described by the subset of Toeplitz matrices [[7]] called convolution matrix:
where h_i are samples of the channel impulse response with length L. Due to the channel influence, received vector $\mathbf{y}$ differs from transmitted vector $\mathbf{x}$. In order to recover the transmitted information, receiver must reverse changes

inferred by the communication channel by means of an equalizer. Thus, the function of the channel equalizer is inversion of the channel matrix $\mathbf{H}$

$$\mathbf{x} = \mathbf{H}^{-1}(\mathbf{y} - \mathbf{w}). \quad (3)$$

$$\mathbf{H} = \begin{bmatrix} h_0 & 0 & \dots & 0 & 0 \\ h_1 & h_0 & \dots & \vdots & \vdots \\ h_2 & h_1 & \dots & 0 & 0 \\ \vdots & h_2 & \dots & h_0 & 0 \\ h_{L-2} & \vdots & \dots & h_1 & h_0 \\ h_{L-1} & h_{L-2} & \vdots & \vdots & h_1 \\ 0 & h_{L-1} & \dots & h_{L-3} & \vdots \\ 0 & 0 & \dots & h_{L-2} & h_{L-3} \\ \vdots & \vdots & \vdots & h_{L-1} & h_{L-2} \\ 0 & 0 & 0 & \dots & h_{L-1} \end{bmatrix}, \quad (4)$$

However, inversion of Toeplitz matrices cannot be performed easily [[7]]. Moreover, estimation of the channel impulse response samples h_i must be performed before equalization. Many time-domain equalizers use least mean squares (LMS) – based algorithm, which iteratively adjusts $\mathbf{H}^{-1}$ filter taps to minimize difference between transmitted and received training samples.

In OFDM [[1]] it is assumed that original transmit block (vector) $\mathbf{X}$ is located in the frequency domain and represents complex amplitudes of N sinusoidal subcarriers. Before transmission into communication channel, symbols from the channel coder are transformed into the time domain by means of inverse discrete Fourier transform (IDFT)

$$\mathbf{x} = \mathbf{T}^{-1}\mathbf{X}. \quad (5)$$

Multi-path propagation, reflection and noise in the communication channel causes distortion of transmitted symbols. Time domain signal at the receiving end of the channel can be described as follows

$$\mathbf{y} = \mathbf{H}\mathbf{T}^{-1}\mathbf{X} + \mathbf{w}, \quad (6)$$

where $\mathbf{X}$ is frequency-domain input vector, $\mathbf{T}^{-1}$ is IDFT matrix. Received vector is transformed back into frequency domain, equalized and passed to the channel decoder. Receiver operation can be described as follows

$$\tilde{\mathbf{X}} = \mathbf{D}^{-1}\mathbf{T}\mathbf{y}, \quad (7)$$

where $\mathbf{D}^{-1}$ is equalizer matrix which can be obtained by inversion of frequency domain channel matrix

$$\mathbf{D} = \mathbf{T}\mathbf{H}\mathbf{T}^{-1}. \quad (8)$$

If channel matrix $\mathbf{H}$ is circulant matrix [[7]], then $\mathbf{D}$ is diagonal, because from (8) follows eigendecomposition of matrix $\mathbf{H}$

$$\mathbf{H} = \mathbf{T}\mathbf{D}\mathbf{T}^{-1}. \quad (9)$$

Since matrix $\mathbf{D}$ is diagonal, its inversion is straightforward.

It is possible to built multi-carrier communication system, where non-sinusoidal carriers are used [[2]], [[5]]. In this case equations (5)-(9) are still intact. The main problem in this case is that matrix $\mathbf{D}$ is not diagonal anymore and its inversion requires additional effort.

For Phi-transform based communication system the unitary transform matrix is defined as follows

$$\mathbf{T} = \prod_{p=l}^{1} \mathbf{B}_p(\phi, \gamma, \psi). \quad (10)$$

In (10) ϕ, γ, ψ are angles and stairs-like orthogonal generalized rotation matrix $\mathbf{B}_p$ is defined as follows:

$$\mathbf{B}_p = \begin{bmatrix} \tau^1_{1,p} & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \tau^1_{2,p} & \dots & 0 & 0 \\ \dots & & \dots & \ddots & \dots & \\ 0 & 0 & 0 & 0 & \dots & \tau^1_{N/2,p} \\ \tau^2_{1,p} & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \tau^2_{1,p} & \dots & 0 & 0 \\ \dots & & \dots & \ddots & \dots & \\ 0 & 0 & 0 & 0 & \dots & \tau^2_{N/2,p} \end{bmatrix}, \quad (11)$$

where τ are elements of unitary four-element single-plane rotation matrix. One variant of this matrix could be:

$$\begin{bmatrix} \tau^1_{q,p} \\ \tau^2_{q,p} \end{bmatrix} = \begin{bmatrix} \mp sin\phi_{q,p} e^{-j\psi_{q,p}} & cos\phi_{q,p} e^{j\gamma_{q,p}} \\ cos\phi_{q,p} e^{-j\gamma_{q,p}} & \pm sin\phi_{q,p} e^{j\psi_{q,p}} \end{bmatrix}. \quad (12)$$

In order to equalize received signal, receiver must perform inversion of Phi domain channel matrix (8). One of the well-known methods to perform matrix inversion is singular value decomposition (SVD) [[8]]. SVD decomposes given rectangular matrix into two orthogonal square matrices $\mathbf{U}$ and $\mathbf{V}$, who share common eigenvalues, and diagonal matrix $\mathbf{S}$ with singular values sorted in descending order

$$\mathbf{D} = \mathbf{U}\mathbf{S}\mathbf{V}^{-1}. \quad (13)$$

Pseudo-inverse of this rectangular matrix

$$\hat{\mathbf{D}} = \mathbf{V}\hat{\mathbf{S}}\mathbf{U}^{-1} \quad (14)$$

now can be found easily since for orthogonal matrices

$\mathbf{V}^{-1} = \mathbf{V}^T$ and $\mathbf{U}^{-1} = \mathbf{U}^T$, where symbol "T" means transposition. Elements of the diagonal rectangular matrix $\hat{\mathbf{S}}$, which is pseudo-inverse of (non square) $\mathbf{S}$, can be found using relation

$$\hat{S}_{k,k} = \frac{1}{S_{k,k}} \qquad k \in 0, N. \tag{15}$$

By summarizing previous equations, Phi-transform based communication system transmitter operation can be described by equation (5). In turn, receiver operation describes equation derived from (7)

$$\tilde{\mathbf{x}} = \mathbf{V}\hat{\mathbf{S}}\mathbf{U}^{-1}\mathbf{T}\mathbf{y}. \tag{16}$$

SVD matrices, necessary for the equalization can be found using (13). Finally we obtain baseband communication system, which is shown in Fig. 1. Zero padding (ZP) block is necessary to mitigate IBI.

IV. CHANNEL ESTIMATION

In order to perform decomposition (13) and, finally, provide equalization, communication channel estimate $\mathbf{D}_{est}$ is necessary. For non-blind channel estimation training sequences must be transmitted. Received training sequences can be compared to original ones and channel influence can be calculated.

Phi domain channel estimate can be obtained by transmitting identity matrix based training sequence. In this case processing at the receive end is very simple, since $\mathbf{D}_{est}$ appears at the output of Phi transform block each time when training sequence is been transmitted. It is worth to notice, that this kind of training sequence is far from optimal, since it is long and has many zeroes. Moreover this training sequence is difficult to use for other purposes such as synchronization [**Error! Reference source not found.**].

Another, more efficient way how to obtain $\mathbf{D}_{est}$ is calculate it from the time domain channel estimate using (8). In this case all classical time domain channel estimation methods are applicable and it is possible to use much shorter training sequences. However, computational complexity of the receiver increases since additionally (8) must be calculated.

There is a big variety of time domain channel estimation methods. We focused our attention just on simplest ones:

1) *Identity (unit) matrix pilot-sequence;*
2) *Delta function pilot sequence;*
3) *Least mean squares (LMS) system identification.*

First, identity matrix pilot sequence based method, implies a similar approach as for time domain estimation. We selected this method in order to compare estimation in different domains.

Second method is based on transmission of discrete counterpart of Dirac delta function. Thus at the receiving side channel impulse response $h(n)$ can be obtained directly. Estimated time domain channel matrix $\mathbf{H}_{est}$ can be constructed by stacking averaged impulse responses into Toeplitz matrix (4).

Third method is much more advanced and is suitable for practical implementations. Channel impulse response is obtained by means of adaptive LMS filter in system identification mode. In this mode filter takes receiver input as a target, whereas pilot blocks are taken as (non-equalized) input. Then time domain channel matrix $\mathbf{H}_{est}$ can be constructed like in previous method.

Computational complexity of researched channel estimation methods is given in Table I. The total computational complexity of Jacobi SVD for square matrix with N rows, in accordance with [9], is $3N^3$. Transform $\mathbf{T}$ generally requires $2N^2$ operations and equalization (14) requires $4N^2 + N$ flops. Therefore total complexity of (16) with M pilot blocks and K LMS updates per block would achieve $3N^3 + 6N^2 + N(1 + K + M) + M$. The most computationally expensive operation is SVD.

TABLE I. COMPUTATIONAL COMPLEXITY OF THE CHANNEL ESTIMATION.

Estimation method	**Complexity, flops**
Identity matrix	0
Delta function, mean (M blocks)	$M(N+1)$
LMS (K updates) + mean (M blocks)	$N(K+M)+M$

V. SIMULATION RESULTS

Computer simulations using Mathworks Simulink environment confirmed viability of the concept. Baseband communication system with perfect timing and sampling frequency synchronization was built for the simulations. Complex constant rotation angle orthogonal transform (CRAOT) [[3]] with $\phi = 30^o$, $\psi = 0$, $\gamma = 90^o$ in (12) was used in a role of Phi transform $\mathbf{T}$. Transmission was carried out in frames consisting of 64 pilot blocks and 20 payload blocks. Zero-padded blocks consisting of 64 payload symbols and 16 padding symbols were used in all simulations. Adaptive LMS filters for both equalization and system identification (estimation) were 8 taps long. Multiple communication systems were simulated and compared. Simulation results in a form of BER plots of communication systems are depicted in fig. 2.

First communication system (Fig. 2, graph A) utilized Phi domain channel estimation and Phi domain equalization. Channel estimation was performed using identity matrix pilot tone structure. This setup requires smallest amount of the computational resources.

In the second experiment (Fig. 2, graph B) we moved estimation into time domain, but leaved estimation algorithm unchanged. From BER graphs it is visible that change of estimation domain almost does not affect communication system performance.

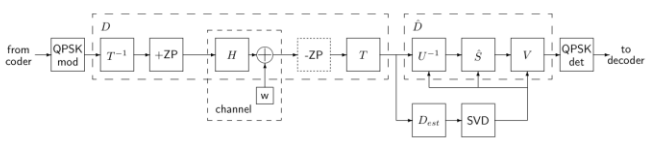

Fig. 1. Baseband communication system with Phi domain channel estimation and Phi domain equalization.

Third communication system (Fig. 2, graph C) exploited different channel estimation algorithm. Unlike previous ones, it did not sample channel matrix directly, but used averaged time domain impulse response for the construction of this matrix. 2dB SNR improvement over previous experiments is achieved due to noise suppression in impulse response averaging process.

Fourth model (Fig. 2, graph D) employed even more advanced channel estimation process based on LMS system identification. This model gives very serious improvement of communication system performance. Communication system achieves BER 10^{-2} at signal to noise ratio 7dB.

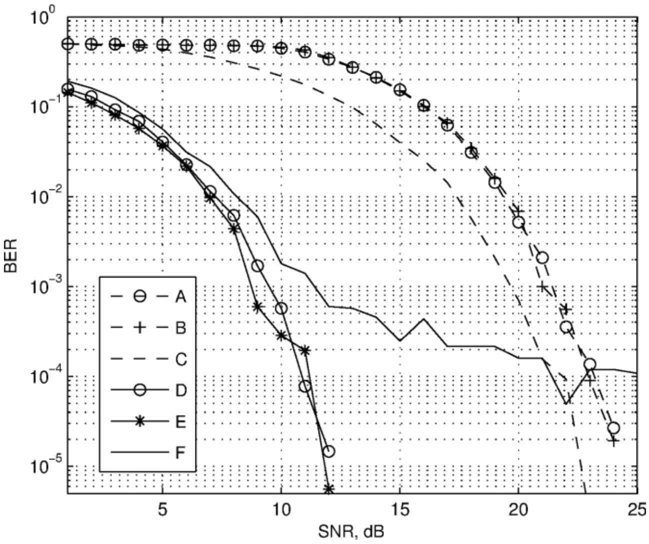

Fig. 2. Performance comparison of estimators and equalizers.

In order to identify impact of the channel estimation on performance of communication system, fifth model (fig. 2, graph E) was created. In this model channel impulse response and, hence, channel matrices (4) and (8) were known to the receiver.

And, finally (fig. 2, graph F), communication system with well-known time domain only LMS equalization algorithm was simulated. This simulation was done in order to obtain reference point for previous graphs. We can see that communication system with LMS estimation and SVD equalization performs better.

VI. Conclusions

Combination of LMS based channel estimation with SVD based equalization provides outstanding equalization for Phi transform based and other MC communication systems. Comparison of BER graphs from fig. 2 and from [[6]] shows that this solution provides approximately 2 dB SNR gain compared to OFDM with classical frequency domain approach.

However, Quality of SVD based equalization strongly depends on quality of the channel estimation. Regarding to this topic, LMS system identification based channel estimation produces almost perfect time domain channel estimate.

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