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Electronic Model of FitzHugh-Nagumo Neuron

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Introduction

Nowadays neural networks are widely used to solve the tasks of signal processing, control and identification, pattern recognition and etc. [1, 2]. Traditional way to investigate the neural network is based on a mathematical model of the network. Despite the powerful computers the modeling of the entire neural network using numerical simulation requires a lot of calculation resources and time. Due to parallel modeling of processes in each of neurons, the neural network model, which consists of electronic models of neurons, should work much faster.

The most famous mathematical model of the real neuron, whose adequacy was validated experimentally, is Hodgkin-Huxley model [1, 3]. But the model of Hodgkin-Huxley is described using four nonlinear differential equations therefore it would be difficult to implement this model into hardware. There are few variants of simplified Hodgkin-Huxley model. The FitzHugh-Nagumo model is one of them [1, 3]. This model is often used for investigation into neurodynamical systems [4–8]. The implementation of the FitzHugh-Nagumo model into hardware is also possible [3–8], so using of this model as a part of the neural network model can be used to increase the speed of modeling.

Analog models of FitzHugh-Nagumo neuron

The set of FitzHugh-Nagumo equations has the form [5]:

$$\begin{cases} \dfrac{dv}{d\tau} = v(v-a)(1-v) - w + I, \\ \dfrac{dw}{d\tau} = \varepsilon(v - \gamma v), \end{cases} \tag{1}$$

where $0 < a < 1$, $\varepsilon > 0$, $\gamma \geq 0$ are dimensionless coefficients and τ is dimensionless time.

The simplest electronic model of FitzHugh-Nagumo neuron is described in [4, 6]. Its circuit is shown in Fig. 1. [6]. Nonlinear part of the FitzHugh-Nagumo equations is formed using nonlinear circuits which consist of in series connected diodes and resistances.

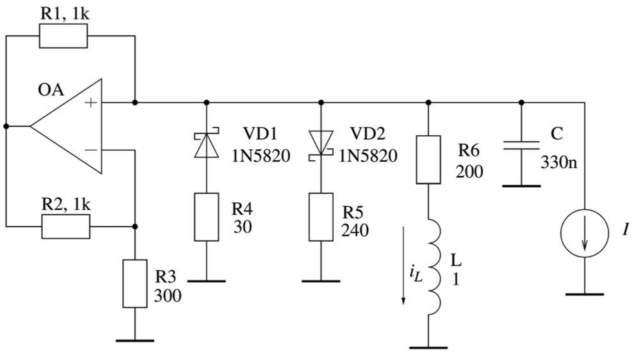

Fig. 1. Circuit diagram of the FitzHugh−Nagumo oscillator [6]

In order to perform analysis of dynamics of the circuit, shown in Fig. 1, the differential equations of this system should be analyzed. If the voltage across capacitance C is denoted V_C, the circuit can be described by equations:

$$\begin{cases} \dfrac{dV_C}{dt} = \dfrac{V_C}{CR_3} - \dfrac{I}{C} - \dfrac{I_d(V_C)}{C} - \dfrac{I_L}{C}, \\ \dfrac{dI_L}{dt} = \dfrac{V_C}{L} - \dfrac{R_6 I_L}{L}. \end{cases} \tag{2}$$

Variables V_C, I_L and I in equation (2) correspond to variables v, w and I in FitzHugh-Nagumo equation (1), while nonlinear function $I_d(V_C)/C$ approximates nonlinear part of FitzHugh-Nagumo system.

For the case I=0 one fixed point is evident: V_C=0 and I_L=0. It is not difficult to express Jacobian at this point

$$J = \begin{bmatrix} \dfrac{1}{CR_3} - \dfrac{1}{C}\dfrac{dI_d(V_C)}{dV_C}\bigg|_{V_C=0} & -\dfrac{1}{C} \\ \dfrac{1}{L} & -\dfrac{R_6}{L} \end{bmatrix}. \tag{3}$$

On the base of given $I_d(V_C)$ approximation [4] or from experimentally measured $I_d(V_C)$ characteristic can be stated that

$$\left.\frac{dI_d(V_C)}{dV_C}\right|_{V_C=0} = 0\,, \qquad (4)$$

so into polynomial form rewritten characteristic equation of the system at the point V_C=0 and I_L=0 is

$$\lambda^2 + \left(\frac{R_6}{L} - \frac{1}{R_3 C}\right)\lambda + \frac{R_3 - R_6}{R_3 CL} = 0. \qquad (5)$$

With values of elements shown in Fig. 1 is not difficult to prove that roots of equation (5) are: $\lambda_1 \approx 10\cdot 10^3$ and $\lambda_2 \approx 100$. Positive roots indicate instability of this equilibrium point.

Verification of the correctness of given theoretical result can be performed using experimental investigation of the electrical circuit shown in Fig. 1 at I=0 or by analyzing set of equations (2) by numerical methods.

Obtained results do not match with the results of experimental investigation of neurons. Without activation of the neurons they are in the rest state, therefore at I=0 they should be represented as stable systems [1, 3].

Finally it should be noted, that in the papers [4, 6] were not presented any expressions, which shows the relation between the FitzHugh-Nagumo equations (1) and values of circuit elements. Without those it is not possible neither design of the circuit for selected coefficients values of FitzHugh-Nagumo equations nor determination of the values of coefficients of the FitzHugh-Nagumo equations for the circuit which is already designed.

Slightly more complex model is presented at [7]. Nonlinear characteristic of the FitzHugh-Nagumo system is approximated by using diodes for switching of linear electric circuits. Experimentally determined accuracy of approximation is also presented.

The attention should be paid to the fact, that nonlinear characteristic realized by model is a little bit simpler than described by the FitzHugh-Nagumo equations. The authors of [7] use v^3 instead of $v^3 - (a+1)v^2$.

Else one of more perspective solutions could be application of programmable structure analog array (FPAA). Due to multipliers, which are integrated into the chip, the mathematically exact realization of the nonlinear equation is possible [8]. Two FPAA chips of type *AN231E04* were used to representation of nonlinear part of FitzHugh-Nagumo equations [8].

The biggest disadvantage of such model is unpopularity of FPAA chips. The first FPAA chip was produced in the year 2000 therefore this technology is rather new and not well-established.

Digital model of FitzHugh-Nagumo neuron

The first equation of the set (1) can be split to linear and nonlinear parts

$$\frac{dv}{dt} = \underbrace{-v^3 + v^2(a+1)}_{i(v)} - av - w + I. \qquad (6)$$

Nonlinear part is denoted as $i(v)$ in expression (6). Performed analysis shows that the greatest problems arise from elaborating analog models of this part of FitzHugh-Nagumo system.

Expression $i(v)$ is polynomial of the third order. It is really difficult to realize such function using analog circuits, however $i(v)$ can be easily calculated using microcontroller. The advantages of realizing $i(v)$ using microcontroller are evident:

1. The program of calculation of the third order polynomial is very simple;
2. Realized function can be flexibly changed.

The proposed method has proper shortcoming – changing signals from analog to digital and back require corresponding converters, therefore the circuit becomes more complex and more expensive. There is no problem with analog-digital converter because all modern microcontrollers have those units, but just a few general purpose microcontrollers have integrated digital-analog converters.

Functional diagram of proposed model is presented in Fig. 2.

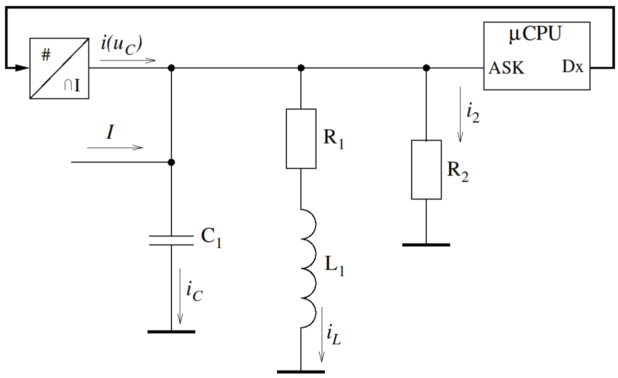

Fig. 2. Functional diagram of FitzHugh-Nagumo model

The model given in Fig. 2 can be described by set of equations:

$$\begin{cases} \dfrac{du_C}{dt} = -\dfrac{1}{R_2 C}u_C - \dfrac{1}{C}i_L + \dfrac{1}{C}I + \dfrac{1}{C}i(u_C), \\ \dfrac{di_L}{dt} = \dfrac{1}{L}u_C - \dfrac{R_1}{L}i_L. \end{cases} \qquad (7)$$

Elements C_1, R_1, L_1 and R_2 are devoted to simulate linear part of FitzHugh-Nagumo equations. Analog-digital converter, which is integrated into microcontroller, converts this voltage to digital form. After that microcontroller calculates nonlinear function $i(v)$ and transfers the result into digital-analog converter. Finally, digital-analog converter converts digitally result of calculation into analog quantity – current i_n.

The system of FitzHugh-Nagumo equations (1) is presented in unitless form, and differs from system of equations (7), which is presented in dimensional form. The relations between corresponding unitless and dimensional variables are

$$v = \frac{u_C}{V};\; w = \frac{i_L}{W};\; \tau = \frac{t}{T}\,, \qquad (8)$$

where V – coefficient with voltage unit, W – coefficient with current unit and T – coefficient with time unit.

Quantities V, W and T should be chosen in that way that the currents and the voltages of the circuit could not exceeded allowable values. The attention should be paid to the fact that nonlinear function $i(v)$ has dimension of current, while its argument has dimension of voltage. In order to express the function and its argument in the same units, the function $i(u_C)$ is rewritten in the form

$$i(u_C) = -k_3 u_C^3 + k_2 u_C^2, \tag{9}$$

where coefficients $[k_3]$=A/V^3 and $[k_2]$=A/V^2.

After choosing the values of V, W and T and knowing the values a, γ and ε from the equations of FitzHugh-Nagumo, the values of the circuit elements are calculated from expressions

$$C = \frac{WT}{V};\ L = \frac{TV}{\varepsilon W};\ R_1 = \frac{\gamma V}{W};\ R_2 = \frac{V}{aW} \tag{10}$$

and coefficients k_2 and k_3 as

$$k_2 = \frac{W(a+1)}{V^2};\ k_3 = \frac{W}{V^3}. \tag{11}$$

Expressions (10) and (11) unambiguously relate coefficients of the FitzHugh-Nagumo equations with the parameters of the model circuit and the function which is implemented into microcontroller. It is noticed, that at known V, W and T the inverse task also can be performed.

Simulation results

Gnucap [8] program was used for simulation. It was chosen due to its similarity to popular Spice program and due to its GNU General Public License.

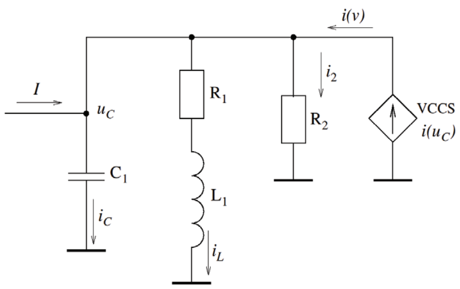

Fig. 3. Simplified circuit diagram of FitzHugh-Nagumo model

Circuit of the model is shown in Fig. 3. The microcontroller and digital-analog converter is replaced with voltage controlled current source (VCCS) which is programmed according (9).

Coefficients of the FitzHugh-Nagumo system are chosen for simulation

$$a = 0{,}95;\ \gamma = 2{,}5;\ \varepsilon = 0{,}005. \tag{12}$$

All coefficients are chosen for obtaining a system which is able to oscillate.

Coefficient V is selected to limit amplitude of voltage u_C up to 5 volts what is a typical value for many into microcontrollers integrated analog-digital converters. Coefficient W is selected to limit amplitude of the currents approximately to 1 mA. Finally the value of coefficient T is chosen to get frequency of oscillations approximately equal 100 Hz. So V=3; W=0,001 and $T = 5\cdot10^{-5}$.

The calculated values of the circuit elements are: C_1=16,67 nF; L_1 = 30 H; R_1=7,5 kΩ ir R_2=3,174 kΩ. Coefficients of (3) are: k_3=3,7037·10^{-5} and k_2=2.167·10^{-5}.

At analysis of obtained circuit elements values and expression (10) the additional shortcoming of the circuit appears: the ratio of capacitor C_1 capacitance and coil L_1 inductance is very small

$$\frac{C}{L} = \left(\frac{W}{V}\right)^2 \varepsilon \approx 5{,}56\cdot10^{-10}. \tag{13}$$

That means choosing of capacitance in the nanofarad range gives inductance of tens Henry range. Situation can be improved by decreasing of T, but it is more rational to replace the coil with active inductor [5].

The transients of u_C at different values of I are presented in Fig. 4.

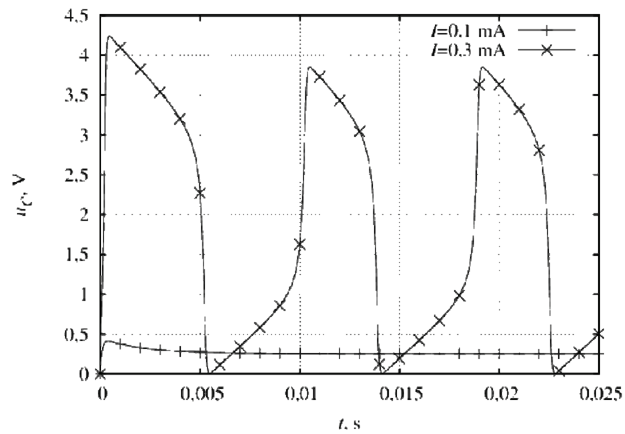

Fig. 4. The transients of u_C at different values of I

The initial conditions were: u_C=0 and i_L=0. It can be seen, that at I=0,1 mA < I_{akt} the current after short settling time reaches its steady state value and no oscillation appears. At the I=0,3 mA > I_{akt} the relaxation oscillations appear. This behavior complies with experimentally determined behavior of real neurons.

Conclusions

By using analog electronic circuits it is difficult to model the nonlinear part of FitzHugh-Nagumo equations. Analog circuits usually are able to represent just simplification of nonlinear part of FitzHugh-Nagumo equations.

Suggested model of the FitzHugh-Nagumo neuron is based on microcontroller, which calculates the nonlinear part of equations mathematically correctly, i.e. without any approximations.

The expressions, which relate the coefficients of the FitzHugh-Nagumo equations, with values of the circuit elements, are also presented.

The biggest disadvantage of the suggested model is the need to use the analog-digital and digital-analog converters.

Dynamic analysis of the circuit shows that the suggested circuit correctly imitates the behavior of the FitzHugh-Nagumo neuron.

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For investigation into neurodynamical systems FitzHugh-Nagumo model is often suggested. One neuron can be easily modeled using numerical methods, but numerical modeling of the entire network of FitzHugh-Nagumo neurons requires a lot of calculation resources and time. To overcome that problem the electronic model of FitzHugh-Nagumo neuron is proposed. The article discusses and compares some electronic models of FitzHugh-Nagumo neurons. The advantages and shortcomings of each model are discussed. Microcontroller based model of the FitzHugh-Nagumo neuron is proposed. The microcontroller calculates the nonlinear part of the FitzHugh-Nagumo equations mathematically correctly, i.e. without any approximations. Simulation results of the suggested model are presented and discussed. Ill. 4, bibl. 8 (in English; abstracts in English and Lithuanian).

A. Petrovas, S. Lisauskas, A. Slepikas. Elektroninis FitzHugh ir Nagumo neurono modelis // Elektronika ir elektrotechnika. – Kaunas: Technologija, 2012. – Nr. 6(122). – P. 117–120.

FitzHugh ir Nagumo neurono modelis dažnai naudojamas neuroninėms sistemoms tirti. Nors vieno FitzHugh ir Nagumo neurono imitavimas skaitiniais metodais didesnių sunkumų nekelia, tačiau, imituojant neuronų tinklą, iškyla skaičiavimo greitaveikos problema, todėl aktyviai kuriami aparatiniai modeliai. Šiame straipsnyje apžvelgiamos ir lyginamos tarpusavyje skirtingų autorių siūlomos schemos FitzHugh ir Nagumo sistemai modeliuoti. Skiriami jų pranašumai ir trūkumai. Pasiūlytas neurono modelis, kuriame netiesinė FitzHugh ir Nagumo sistemos dalis kuriama skaitmeniškai ir todėl matematiškai tiksliai. Pateikiami siūlomo modelio imitavimo rezultatai. Il. 4, bibl. 8 (anglų kalba; santraukos anglų ir lietuvių k.).