Cardio Signals Extraction Based on Finite Sums of Exponential Functions

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Introduction

For many applications a well known problem is an extraction or equivalent estimation of some predefined signal or component form at time series contaminated by noise [1]. Andrew Harvey and Giuliano De Rossi in signal extraction book charter provided the basis for fitting cubic splines to cross sectional data [2]. They work on signal extraction in nonlinear models and have focused on the development of particle filtering. This appears to offer a viable simulation based approach in many situations.

Cardiac signals are not casual generated signals, i.e., the signal could be predicted in time. Understanding the functional processes of the heart and especially interaction between various types (electrical, mechanical, hemodynamic) processes in the heart is still very problematic and heavily tackled task. To find a relatively new tool with the purpose of mapping the specific regions in the heart is the main idea of this paper. Signals identification using the proposed structure based on Hankel rank function employment is presented. In this paper there are shown the results of cardio signals extraction using the proposed structure based on employment of sum of exponential functions.

The work is divided in four sections. The first theoretical section describes the mathematical reasoning of the evaluation of Hankel matrices for system identification. The second section is dedicated to cardiac signals. The third section is showing the computational results. Conclusions delivered in the last section.

Hankel matrices for system identification

Usually, in system identification Hankel matrices are formed when given a sequence of output data and a realization of an underlying state-space or hidden Markov model is desired [3], but in this paper the ranks of the Hankel matrix will be used as features for the system identification purposes. Let us assume that these signals are sequence of data samples \( \{x_j, x_{j+1}, \ldots, x_N\} \), where \( N \) is an even number of data samples.

The calculation scheme of the signal reconstruction executes four steps. The accuracy level \( \varepsilon \) is fixed in the first step [5]. Hankel rank matrix estimation \( K \) and parameters \( \mu, \rho \) are computed in the second step [4, 5]. In the third step the samples of signal are reconstructed basing on the sum of exponential functions [6]. If there are \( M \) measurements in one second, then \( h = 1/M \). The number of samples depends on condition: \( \text{err} < \text{standard} \), where err defines the difference value between given set of reconstructed and real signal.

Calculation scheme

1. Choose the accuracy level \( \varepsilon \);
2. Find Hankel matrix \( H \) rank estimation that satisfy the condition and compute the parameters \( \mu, \rho \);
3. Reconstruct \( K \) samples of the signal \( \{x_1', \ldots, x_N'\} \) based on the sum of the exponential functions
   \[
   x'_j := \sum_{r=1}^{K} \mu \cdot \rho^r \cdot e^{r \cdot j \cdot h}
   \]
4. Select \( M_1 \) and \( M_2 \), which satisfy the condition \( \text{err} < \text{standard} \) and difference between \( M_1 \) and \( M_2 \) is the biggest
   \[
   \text{err}_{M_1 M_2} := \frac{1}{M_2 - M_1} \sum_{j=M_1}^{M_2} (x'_j - x_j)^2
   \]
   \[
   \text{err}_{M_1 M_2} < \text{standard}.
   \]

M1 and M2 are selected such, which satisfy the condition \( \text{err} < \text{standard} \) and difference between M1 and
M2 will be the biggest. The reconstructed signal \((x_1',\ldots,x_N')\) is acquired using operations with Hankel matrix rank estimation and compared with real signal \((x_1,\ldots,x_N)\), where \(N\) is the even number of samples in the signal.

**Information about signals**

Three cardio signals were analyzed: electrocardiogram (ECG), which reflects the electric heart activity; impedance cardiogram (ICG) - reveals the haemodynamic properties of the cardiovascular system; seismocardiogram (SCG) that shows the changes of heart mechanic activity. These three signals describe the activity of person heart from three different sides. ECG, ICG and SCG were recorded synchronously, but segments of the synchronization issue will not be analyzed.

In the same way the ICG and SCG signals were expressed also.

The signals were recorded and analyzed by means of Multi cardio signal analysis system developed in the Kaunas Institute of Cardiology and produced by “Kardiosignalas” Ltd. (Kaunas, Lithuania). The system hardware consists of notebook, sensors and 16 channel recorder for synchronous recording 12 lead ECG, ICG and SCG with sampling frequency \(F_s=1/h=2000\) Hz. The software contains programs for input, filtering of signals, recognition characteristic points and measurement of parameters. All signal analysis techniques used in this paper are implemented on a PC using custom software developed in Matlab R2007b.

One of study tasks was to compare the complexity of cardiac signals recorded for diseased and healthy persons. For this purpose the test data base consisting of 78 patients (42 males and 36 females, mean age 58±13) with ischemic heart disease and 7 healthy persons (6 males and I female, mean age 55±12) was created and used.

The computational results of applied mathematical reasoning are shown in the next section.

**Computational results**

As was mentioned, it is possible to construct Hankel matrix of QQ and describe ECG, ICG or SCG signal with Hankel matrix rank estimation and divided them to segments with the same complexity, i.e., map the specific region based on employment of sum of exponential functions.

![Fig. 1. Data samples selected from the ECG, ICG and SCG signals according to QQ intervals](image1)

![Fig. 2. The alteration of SCG segments number when standard \(-0.001\)](image2)

QQ lengths are different. They depend on situation, age of patient, history and other clinic symptoms. On purpose to evaluate the sufficient calculating limits of parameters, two patients with the longest (1960 deduction) and the shortest (1575 deduction) QQ intervals were compared.

![Fig. 3. The alteration of SCG segments number when standard \(-0.01\)](image3)

Mathematical model of craniocaudal forces developed by a mechanical activity of heart and blood flow through great arteries has been described in many papers [7]. QQ of SCG signal defined by segments is showed in the Fig. 2, where segments are on the Y axis, and the level of \(\varepsilon\) of H rank estimation - on the X axis.

The main idea is that signal splitting into segments is not depended on the length of signal. Fig. 2 and Fig. 3 show that the results with various \(\varepsilon\) are different. It is known that the higher rank value (with smaller \(\varepsilon\)) describes higher signal complexity in certain interval [8]. From numerical relation between \(\varepsilon\) and the standard it could be clearly manifested that the number of segments is smaller with smaller \(\varepsilon\) value. The less accuracy is required for the
reconstructing task. Moreover when higher $\varepsilon$ value is used, the higher reconstruction accuracy is defined. Therefore more segments are used in this case. The difference between used segments for the reconstruction of shorter and longer QQ, is small due well defined parameters – $\varepsilon$ and standard.

ICG signal depends on some body processes, such as breathing, physical readiness or muscle tone, etc. An ICG signal consists of two major components: one, reflecting the respiratory movements and another, reflecting the blood flow in the chest.

However, the usual way based on frequency range separation of these two components is not optimal for this signal and does not allow performing the detail analysis [9].

Fig. 4 and Fig. 5 show the alteration of ICG segments number with standard – 0.001 (Fig. 4) and standard - 0.01 (Fig. 5). Number of segments is smaller with smaller $\varepsilon$ value too. Despite the fact, that QQ interval of ICG are different lengths, both the longest and shortest QQ interval with standard 0.01 and epsilon -6 or -8, it is possible to divide into 41-42 segments with standard -0.01 and $\varepsilon$ -10^-6 or 10^-8.

Fig. 6 show that with standard – 0.001 the number of the shortest or the longest QQ interval of ECG number of segments depends on $\varepsilon$ and varies 400 to 580, 500 to 700 or 790 to 940 segments.

Regardless of the QQ interval length, QQ intervals of ICG and SCG signals were divided into constant intervals with standard 0.01 and $\varepsilon$ = 10^-6 (Fig. 3, Fig. 5). ECG signal is more complicated. Fig. 7 shows, that rather big standard (0.01) does not help to split different length QQ intervals to constant number of segments.

Conclusions

Reconstruction algorithm that incorporates the standard value and quality measure is presented in this work. The experimental investigations have shown that the amount of segments of all measured signals is different because every signal and length of QQ is unique to each person. It is promising method which allows analyzing the cardio signal, expressed by limited number of parameters. The results show that expressing of cardio signals with Hankel matrix and mapping the specific region based on employment of finite sums of exponential functions could be useful for future diagnostic purposes and classification of “healthy” and “sick” persons groups. Future work will involve the incorporation of adaptively changing length of segment which allows significantly reduce the amount of parameters.

References


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The aim of the work was to adapt the analysis of the ranks and mapping the specific region of QQ intervals based on employment of finite sums of exponential functions to three synchronously recorded cardio signals – electrocardiogram (ECG), impedance cardiogram (ICG) and seismocardiogram (SCG). Reconstruction algorithm that incorporates the standard value and quality measure is presented in this work. The results show that expression of cardio signals by Hankel matrix and mapping of the specific region based on employment of sum of exponential functions could be useful for development the diagnostic technologies in future. Ill. 7, bibl. 9 (in English; abstracts in English and Lithuanian).


Darbo tikslas buvo pritaikyti rankų analizę bei signalų vertinimą baigtinėmis eksponenčių sumomis, palyginant tris sinchroniškai užregistruotus širdies elektrinius signalus – electrocardiogramą (EKG), impedanskardiogramą (IKG) bei seismokardiogramą (SKG), suskaidant juos QQ intervalo ilgio atkarpomis. Šiame darbe pateiktas rekonstrukcijos, t. y. signalo vertinimo, algoritmas įvertina atkūrimo kokybę ir standartą. Gauti rezultatai rodo, kad širdies signalų išreikšimas Hankelio matricos rangu bei segmentų radimas, vertinant signalą baigtinėmis eksponenčių sumomis, tolesniuose tyrimuose gali padėti tobulinti diagnostines technologijas. II. 7, bibl. 9 (anglų kalba; santraukos anglų ir lietuvių k.).