Finite Element Analysis and Structural Optimization of a Permanent Magnet Spherical Actuator

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Introduction

In recent years, the motors and actuators with three degrees-of-freedom have attracted special interests as novel direct drive type actuators for many modern devices applications, such as robotic joints, computer vision, transporting elements and omnidirectional wheels etc. The spherical actuator as one of them can provide advantageous features over traditional drive mechanisms which are usually constructed by several conventional drive motors or actuators, each having one degree of freedom and reducing the position accuracy, stiffness, dynamic performance and efficiency of the system [1–7]. Unlike the conventional cylindrical motor, the permanent magnet spherical actuator (PMSA) can implement 3 degrees-of-freedom motion and rotate with any axis in space. The electromagnetic field distribution and torque characteristics are important aspects for application and research, the rational torque analysis can provide the bases for structural optimization and control system modeling and application.

Genetic algorithm (GA) is a stochastic and parallel search technique based on the mechanisms of natural selection, genetics and evolution, which was first developed by Holland in 1970s [8-10]. GA is known to be a powerful tool for performing search in complex spaces. In recent years, GA has been widely applied to different areas such as fuzzy systems, neural networks etc. The only drawback is that when dealing with multi-modal functions with peaks of unique value, ordinary GA is characterized by converging to the best peak of the space (or to a space zone containing several other best peaks) and to lose adequate individual sampling of other peaks in other space zones. This is called the genetic drift and is not a correct behavior for many kinds of problems in which other locations of functions’ optimal values are more interested to know. The niche and species concepts have been introduced for overcoming this behavior [11]. Niche is viewed as an organism’s task in the environment and species is a collection of individuals with similar features.

In this way, its main purpose is to form stable subpopulations of organisms surrounding separate niches by forcing similar individuals to share the available resources. The niche genetic algorithm aims at gathering the individuals on several peaks of fitness function in the population according to genetic likeness and they permit GA to investigate those peaks in parallel. The fittest individual in the niche is kept unchanged or with high fitness value, while others in the niche are changed to reduce their fitness values sharply. So the individuals in the population may be dispersed into the whole search space. Thus some diversity can be maintained effectively during the generations in the population. The key problem for solving in the optimization is to avoid falling into local minimum, improve the convergence speed. Thus, new optimization techniques are used to solve the constrained problem.

3D electromagnetic systems are the core elements of many electric devices such as motors or actuators. The main indexes such as efficiency, force or torque capability and other performances can be optimized by some schemes. However, compared to the conventional motors or actuators, the three dimensional structures of PMSA make the structure optimization process more complicated and hard to implement.

The aim of this study is to calculate and evaluate a novel PMSA by 3D finite element analysis of electromagnetic field and torque characteristics, also with detailed discussion on the effects of some key factors, then the improvement on the niche genetic algorithm for optimization applications is performed by using the combination of sharing method and the exclusion mechanism to derive better effects. The actual situations of PMSA optimal design are introduced with application of niche genetic algorithm to meet the final practical requirements. The work results can have maximum capabilities for further optimization and control system design of this kind of actuator with multi-degrees of freedom.
3D finite element analysis of PMSA

The main structure of the actuator is composed by two layers of eight stator poles and a sphere permanent magnet rotor. Figure 1 shows the stator and rotor models and schematic view of the whole PMSA. The sphere PM rotor is divided into four parts with N-S pole alternately distribution. Each pole can be magnetized by radial or axial direction. The motor has a simple structure, reliable performance advantages and its main structure parameters can be shown as follows

Table 1. Structure parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of upper layer coils</td>
<td>25mm</td>
</tr>
<tr>
<td>Radius of lower layer coils</td>
<td>15mm</td>
</tr>
<tr>
<td>Radius of coils</td>
<td>20mm</td>
</tr>
<tr>
<td>Air-gap length</td>
<td>1mm</td>
</tr>
<tr>
<td>Radius of permanent magnet</td>
<td>50mm</td>
</tr>
<tr>
<td>Magnet relative permeability</td>
<td>1.0445</td>
</tr>
</tbody>
</table>

According to the extremum theory, the functional problem is equivalent to the following equations:

\[
\frac{\partial F}{\partial A_{xi}} = \frac{\partial F}{\partial A_{yi}} = \frac{\partial F}{\partial A_{zi}} = 0,
\]

then the finite element equation can be finally established as

\[
[K][A] = [P],
\]

where \([K]\) and \([P]\) are coefficients matrix by elements.

Like the conventional reluctance motors, the operation of the PMSA is also based on the reluctance forces. The difference is that the flux path is not closed due to the non-magnetic material used in the rotor. It can be considered that the torque or force exerted on the rotor is the summation of individual rotor/stator pair interactions and the torque calculation model has the linear property.

The PMSA’s output torque about the axis \(k(k_x, k_y, k_z)\) is given by [13–15]

\[
T_k(i, \theta) = \frac{\partial W_i(i, \theta)}{\partial \theta_k},
\]

where \(i\) denotes the input current vector, \(\theta\) is the parametric vector of the orientation of the rotor.

Using the FEM software, the setup of solution model can be operated as follows:

a. Input the configuration parameters, basic sizes to the 3D module, the software will automatically construct the finite element geometric model as shown in Fig. 1. Due to the symmetrical feature, the modeling can also be performed by rotating stretching 2D model;

b. Define the properties of materials in the project menu, special attention should be paid to the NdFeB by correctly setting the direction of the magnetization, as shown in Fig. 2;

c. Load excitation and boundary conditions;

d. Set solution area. In the calculation of the actuator torque by 3D FEM, the computational domain should be selected large enough to be possible to obtain relatively accurate data. The stator and rotor geometry for the software itself can be used adaptive subdivision. The actuator can implement free ±180° spin rotation, deflected ±45° nutation. The excitation source parameters and the starting to terminal positions of the rotor motion as show in Table 2.

The total energy functional \(F(A)\) can be calculated as the summation of every element

\[
F(A) = \sum_{i=1}^{n_0} F_i(\hat{A}).
\]
At the initial position, 1000 Ampere-coil current is applied to the A-A1 coil to pulse the rotor move in the $\alpha$ direction with $0^\circ \sim 45^\circ$. When the action of A-A1 finished, coil B-B1 is imposed with 1000 Ampere-coil current to the next action. The same activated combination of coils can implemented the rotor along with $\theta$ direction. In the condition of one stator pole activated, the radial magnetized and the axial magnetized magnets produced torque maximum value making the rotor spin with 90 degrees.

The spherical NdFeB magnetic flux density can reach the peak modulus 1.2T. Circumference can also be seen from the Fig. 3, the magnetic flux density modulus values exhibit great regularity. Within the 360-degree rotation of the permanent magnet, the rotor magnet flux density modulus has 4 cycles. Circumference modulus flux density peak values are in the N pole and S pole alternately places.

Ideal spherical NdFeB and the stator coil can reach a very small distance and the spherical volume is with the largest value, so the modulus peak torque up to 0.83Nm, as shown in Fig. 5. The air gap is uniform between the spherical NdFeB rotor and stator and thus the change of torque modulus is relatively smooth.

<table>
<thead>
<tr>
<th>Stator coil current Ampere-coil</th>
<th>Spherical rotor initial position</th>
<th>Spherical rotor current Ampere-coil</th>
<th>Spherical rotor position after motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>I_A=0, I_A1=0</td>
<td>$\theta=0^\circ$, $\alpha=0^\circ$</td>
<td>I_B=1000, I_B1=1000</td>
<td>$\theta=0^\circ$, $\alpha=45^\circ$</td>
</tr>
<tr>
<td>I_B=0, I_B1=0</td>
<td>$\theta=0^\circ$, $\alpha=45^\circ$</td>
<td>I_C=1000, I_C1=1000</td>
<td>$\theta=0^\circ$, $\alpha=90^\circ$</td>
</tr>
<tr>
<td>I_C=0, I_C1=0</td>
<td>$\theta=0^\circ$, $\alpha=90^\circ$</td>
<td>I_D=1000, I_D1=1000</td>
<td>$\theta=0^\circ$, $\alpha=135^\circ$</td>
</tr>
<tr>
<td>I_D=0, I_D1=0</td>
<td>$\theta=0^\circ$, $\alpha=135^\circ$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2. Stator and rotor models of the actuator

Fig. 3. The ideal spherical permanent magnet circle magnetic flux density modulus

Fig. 4. The ideal magnetic field strength line of the rotor

Fig. 5. Torque characteristics of PMSA

The PMSA has the specialties on the structure and usage, such as the spherical rotor shape, opening holes for output axis. The analysis model is complicated with poor forms of functions used; some design variables have the discrete requirements, resulting in hard implementation of the optimization compared with the conventional motors. The optimization mathematical model of PMSA contains optimal design variables, constraints and object function.

**Niche genetic algorithm with its improvement**

In biology, the environmental niche refers to a specific process in the evolution of a particular environment. Genetic algorithm simulation methods are mainly based on the pre-selection mechanism, the crowding out mechanisms and the sharing act of choice strategies, whose goals are maintaining diversity populations and avoid prematurely eliminating excellent individuals within the populations.

In the improved Niche algorithm, firstly to select the effective evolutionary $M$ samples, then calculate the fitness function of each individual’s size and the average fitness value, and select the memory of the previous $N$ samples of $M$ individuals by random walk sampling methods, from the population to select a certain number of individuals as parents to reproduce. In the crossover and mutation operations, the fuzzy logic controller is used for adaptive crossover probability and mutation probability with the standard of average fitness: when the average fitness and best fitness are in large differences, indicating that the average life fitness is relatively small, the crossover probability and mutation probability should be chosen larger. The ambiguity is determined by the maximum...
fitness function and the minimum fitness function interval
\[ t = \frac{f - f_{\min}}{f_{\max} - f_{\min}} \] as standard.

After the high-fit operation, some niches can be conducted out, getting \((M+N)\) individuals of new species. To these individuals, solve the Hamming Distance
\[ \|X_i - X_j\| = \sqrt{\sum_{k=1}^{M+N} (x_{ik} - x_{jk})^2} \quad (i = 1, 2, ..., M+N; j = 1, 2, ..., M+N). \quad (7) \]

During the iteration process, select the elitist conservation groups’ strategy. When \(\|X_i - X_j\| \leq L\), the values of individual numbers and individual fitness are compared. The individuals with small fitness are processed by penalty function
\[ F_{\text{max}}(X_i, X_j) = \text{Penalty} \quad (8) \]
to make the punished individuals eliminated out with higher possibility; certain distances are hold between the individuals to enhance the diversity and lessen the prematurity phenomenon
\[ S' = \{s_k^*: f(s_k^*) = \max(f(S_k)), s^*_k \in S_k, k = 1, 2, ..., m\}, \quad (9) \]
where \(S_k\), \(S_k^*\) are denoted as the best individuals of the \(k\) and \(k+1\) generation groups of \(s_k\) and \(s_k^*\), \(f(s_k^*)\) are the corresponding fitness value.

Genetic algorithm has inadequate aspects itself: inadequate numbers of samples are easy to fall into local optimum, which is like a premature, so the end result sometimes can not reach the expectations. The computing niche algorithm has previously given up a large part of the samples.

According to the ordinary GA, the more number of individuals, the little possibility of falling into local optimal solutions. Also combined with using the niche, it is necessary to eliminate particularly large number of individuals for further optimal choice. The whole process can be subdivided into following steps:

Step1. Initialize the parameters.
Step2. Produce the initial population groups with evaluations.
Step3. The initial population groups are equally divided into several sub-populations and sub-populations evolve independently.
Step4. Choosing within the shared groups.
Step5. Select a certain number of good individuals by corresponding ratio form the sub-populations, and then calculate the Hamming Distance between every two individuals.
Step6. Regulate the fitness functions between the groups.
Step7. According to the adjusted fitness of the population groups, new sub-groups are reselected and produced.
Step8. Sub-groups evolve independently.
Step9. Remove the worst performing sub-groups with successive generations of solution groups, and generate new sub-groups of the same size.

Step10. Determine the convergence, if the convergence conditions are met or evolved desired generations, the evolutionary process terminates.

In the design of traditional single degree of freedom motor optimization process, the different optimization purposes to selecting the optimal objectives are not very consistent. For example, the efficiency, power factor, highest starting thrust force, lowest cost, minimum usage of permanent magnets etc. can be chosen as the optimization objects. Since the PMSA is very immature technology, it is unreasonable to take the efficiency as the preferred objective. Taking into account this factor, the output torque and material consuming are set as the optimization objectives.

The constraints on the optimal design of PMSA are mainly the performance constraint, the configuration constraint and the cost constraint. The special applications of PMSA make it be paid more attentions on the torque output ability, the interaction force between the rotor and the activated stator. The constraints equations can be shown as:
\[
\begin{align*}
&g_1(x) = (T_\alpha - T_{\beta})/T_{\beta} \leq 0, \\
&g_2(x) = (T_\gamma - T_{\beta})/T_{\beta} \leq 0, \\
&g_3(x) = (T_\kappa - T_{\alpha})/T_{\alpha} \leq 0, \\
&g_4(x) = (T_{\alpha} - T_{\beta})/T_{\beta} \leq 0, \\
&g_5(x) = (F_{\alpha} - F_{\beta})/F_{\beta} \leq 0, \\
&g_6(x) = (F_{\gamma} - F_{\beta})/F_{\beta} \leq 0, \\
&g_7(x) = (F_{\beta} - F_{\kappa})/F_{\kappa} \leq 0, \\
&g_8(x) = (F_{\kappa} - F_{\kappa})/F_{\kappa} \leq 0.
\end{align*}
\quad (10)\]

The configuration constraints equations based on the rotor and stator size can be given as:
\[
\begin{align*}
&0 < R_s \leq 2R, \\
&0 < H_s < 2R, \\
&8\times 4\pi R_{\theta}^2 < 4\pi(R_s + \sigma)^2, \quad (13)
\end{align*}
\]
where \(R_s\) is the radius of opening hole; \(R\) is the radius of rotor; \(H_s\) is the length of opening hole; \(R_{\theta}\) is the upper radius of stator; \(R_s\) is the inner radius of stator; \(\sigma\) is the length of air gap.

For choosing the optimal design variables, the most direct parameters related to the torque and material consumption should be chosen, i.e. the public variables \(\sigma\), \(H_s\) and non-public variable \(R_{\theta}\) with the premise for determining the dimensions of \(R\), \(R_{\theta}\) and \(R_{\theta}\). The optimization variables can be expressed as
\[ X = [X_1, X_2, X_3]^T = [\sigma, H_s, R_{\theta}]^T. \quad (14) \]

So the optimization function of torque can be shown as
\[ f(X) = \frac{1}{T(X)}. \quad (15) \]
The optimization function of effective material function can be shown as

\[ \text{Cost}(X) = \sigma_{\text{Cu}}(V_{\text{Cu}} - V_{\text{NdFe}}) Y_{\text{Cu}} + \sigma_{\text{NdFe}}(V_{\text{NdFe}} - V_{\text{NdFe}}) Y_{\text{NdFe}} \],

(16)

where \(\text{Cost}(X)\) is effective material function of stator and rotor, which is a function of the configuration parameters; \(\sigma_{\text{Cu}}\), \(\sigma_{\text{NdFe}}\) are the densities of copper and rare earth material respectively; \(V_{\text{Cu}}\), \(V_{\text{NdFe}}\) denote the volumes of copper and rare earth material respectively; \(Y_{\text{Cu}}\), \(Y_{\text{NdFe}}\) are the prices of copper and rare earth material respectively.

Table 3. Initial Calculation Datum of PMSA before Optimization

<table>
<thead>
<tr>
<th>Stator current</th>
<th>Initial datum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tx</td>
</tr>
<tr>
<td>800 Ampere-coil</td>
<td>-0.015195</td>
</tr>
<tr>
<td>900 Ampere-coil</td>
<td>-0.015341</td>
</tr>
<tr>
<td>1000 Ampere-coil</td>
<td>-0.015487</td>
</tr>
<tr>
<td>1100 Ampere-coil</td>
<td>-0.015634</td>
</tr>
</tbody>
</table>

Table 4. Calculation Datum of PMSA after Optimization and Comparison

<table>
<thead>
<tr>
<th>Stator current</th>
<th>Optimized datum by the improved niche genetic algorithm</th>
<th>EnhancementT/%</th>
<th>Enhancement Mag(F)/%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tx</td>
<td>Ty</td>
<td>Tz</td>
</tr>
<tr>
<td>800 Ampere-coil</td>
<td>-0.09021</td>
<td>-0.54975</td>
<td>-0.042264</td>
</tr>
<tr>
<td>900 Ampere-coil</td>
<td>-0.09026</td>
<td>-0.5958</td>
<td>-0.04239</td>
</tr>
<tr>
<td>1000 Ampere-coil</td>
<td>-0.09033</td>
<td>-0.64186</td>
<td>-0.042517</td>
</tr>
<tr>
<td>1100 Ampere-coil</td>
<td>-0.09039</td>
<td>-0.68792</td>
<td>-0.04264</td>
</tr>
</tbody>
</table>

Based on the previous set optimization objects, the final optimization object function of PMSA is

\[ \text{MaxF}(X) = F(T, \text{Cost}) = \frac{T}{\text{Cost}}, \]

(17)

where \(T\) is the whole thrust torque of rotor; \(\text{Cost}\) is the material consumption as mentioned above. All of the joint equations constitute the constraint optimization mathematical model of PMSA.

Using the improved niche genetic algorithm, a multi-degree of freedom PMSA is optimized. The effectiveness of the method can be directly determined by the changes in output torque characteristics whether meets the requirements. Table 1 shows the calculation datum before the optimization process. Table 2 gives the calculation datum after the optimization process and the comparison between the initial and the optimized effects. It can be noticed the output torque has been enhanced obviously by 9.03%, 8.23%, 7.57%, and 7.02% respectively. The detailed configuration parameters’ changes are given in table 3. Table 4 shows the comparison on material consumption between the original and optimized schemes. From table 3 to 6, the weight of stator and rotor decreased by the proposed optimization process with the premise of all indexes can meet the constraints. The application of niche genetic algorithm not only can improve the torque to meet the objectives, but also can effectively reduce the amount of material.

Table 5. Comparison of the Optimization Variables

<table>
<thead>
<tr>
<th>Terms</th>
<th>Initial Scheme</th>
<th>Optimization Scheme</th>
<th>Change Amplitude (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air-gap length (mm)</td>
<td>1</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>Inner radius of stator (mm)</td>
<td>40</td>
<td>41</td>
<td>1</td>
</tr>
<tr>
<td>Open hole length (mm)</td>
<td>10</td>
<td>12</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 6. Comparison of the Effective Material Consumption

<table>
<thead>
<tr>
<th>Terms</th>
<th>Initial Scheme</th>
<th>Optimization Scheme</th>
<th>Saving (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM (kg)</td>
<td>3.8494</td>
<td>3.8447</td>
<td>12</td>
</tr>
<tr>
<td>Copper (kg)</td>
<td>0.9431</td>
<td>0.9150</td>
<td>2.98</td>
</tr>
<tr>
<td>Total Weight (kg)</td>
<td>11.3996</td>
<td>11.1643</td>
<td>2.06</td>
</tr>
</tbody>
</table>
Conclusions

This paper presents the finite element analysis and application of new improved niche genetic algorithm for electromagnetic system optimization of PMSA. The spatial magnetic field model, torque calculation model and 3D finite element application are presented and developed. According to the special configuration, the rotor output torque and material consumption are chosen as optimization objects during the optimization process, while keeping several parameters constant such as the rotor radius, opening hole radius, stator upper and lower radius. The ordinary genetic algorithm is improved during the optimization process with enhanced torque and reduced effective material, which meet the desired objective functions’ requirements through comparison and proved the effectiveness of the optimization method.

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References


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In order to derive the characteristics and adjust the electromagnetic system to the ultimate purpose of achieving better torque output and material usage reduction with constraints, this paper presents the finite element analysis and using improved niche genetic algorithm to the optimal design of a permanent magnet spherical actuator. The magnetic field distribution and torque calculation model based on finite element software are introduced and discussed. The proposed optimization algorithm surmounts effectively the local convergence problem of standard genetic algorithm. The sharing-between-population mechanism is proposed by means of using the better factor of population, saving best result strategy and enhancing global and partial searching ability for earlier achievement of optimal solution. The results show the output torque has been increased and the material has been effectively saved by comparison, which provides the references for related problems. Ill. 5, bibl. 15, tabl. 6 (in English; abstracts in English and Lithuanian).


Siekiant padidinti elektromagnetinių sistemų sukimo momentą atliktas baigtinių elementų tyrimas ir sferinės pavaro su nuolatinio magnetu struktūros optimizavimas taikant genetičią algoritmą. Sudarytas modelis, skirtas magnetinio lauko pasiskirstymo įvertinimui ir sukimo momento apskaičiavimui taikant baigtinių elementų metodą. Nustatyta, kad sukimo momentas padidėjo. IL. 5, bibl. 15, lent. 6 (anglų kalba; santraukos anglų ir lietuvių k.).