Modeling of Steady State Operation of Two-Input DC-DC Converter for Combining of Harvested and Reserve Powers

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Abstract—Modeling of the designed two input DC-DC converter for combining harvested and reserve powers is presented in the paper. Analytical model of the converter in steady state conditions was derived using differential equations averaging method. The results agreement comparing with the converter circuit simulation in LTSPICE software is shown. Analytical expression between control variables (switching duty cycles) and independent quantities (sources’ voltages and load current) is derived. A converter controller algorithm seeking to minimize the power drawn from the reserve source is suggested.

Index Terms—Pulse width modulation converters, modeling, circuit analysis, photovoltaic cells.

I. INTRODUCTION

The electric energy generated by renewable sources (photovoltaic, wind, hydro, etc.) most often has to be converted to the electrical energy having voltage and current compliant to the requirements of an electrical load. A variety of electronic power converters (DC-DC, DC-AC, AC-DC) are used for this purpose [1]. It became common that these converters are also responsible for combining and shaping energies generated by the several renewable sources and/or energy storage batteries in order to supply the load. In this paper we consider the system configuration shown in Fig. 1.

Fig. 1. Power supply from two sources via two-input converter.

The reserve source \(U_2\) supplies the energy from the power grid. This is the energy that a consumer is billed for. We assume that \(U_2\) is always capable of supplying the power demanded for the normal load operation. The energy generated from the renewable source, for example solar panel, is fluctuating over the day and year season. It is possible that instantaneous power drawn from the source \(U_1\) is not sufficient to fully supply the load. The proposed setup is aimed to minimize energy consumed from the reserve source \(U_2\) as a result of some portion of energy taken from the source \(U_1\). Such a configuration could be suitable for loads having DC plug for power supply and a solar panel mounted individually at the consumer side. The conversion from DC (from renewable source) to AC, connecting AC to the power grid or wiring new AC lines to the load location are the difficulties that may be avoided using the proposed setup.

In this paper we target some design issues of the two-input DC-DC converter. The input voltages \(V_1\) and \(V_2\) in general are not equal. Even more voltage \(V_1\) is not fixed due to the varying availability of the renewable energy. We also assume that the load voltage \(V_0\) is equal to \(V_2\), since \(V_2\) is the output of the original DC power supply of the load (laptop computer, TV set-top-box, charger, antenna amplifier, etc.). We will present a steady state analysis of the suggested converter. It is expected that the results of this analysis will support the possibility of implementing the DC-DC converter for combining power generated by the renewable and reserve sources. In the future definition of the converter’s dynamic characteristics will also be needed for the design of closed-loop control of the converter. Meanwhile, we focus on electronic schematics without a feedback controller.

II. STRUCTURE OF THE CONVERTER

Many publications report investigations of the so called current sharing techniques and structures [2], [3]. However, the most often authors deal with the problem of ensuring current balance as in the implementation of redundant power supplies [4]. In our case it is the opposite, i.e. consumption of the energy from the renewable source is the priority, while the remaining energy should flow from the reserve source. Some authors have already presented similar approach using conversion to AC and/or using energy storage batteries [5], [6]. In order to minimize the cost of the converter we attempt to avoid batteries and conversion to AC. In addition to operating two power supplies in parallel the converter must ensure voltage level conversion. Therefore, we expect that switching DC-DC converter with two inputs should serve for the purpose (Fig. 2). PWM control signals of switches \(S_1\), \(S_2\) and \(S_3\) are shown in Fig. 3. It is essential that
the switches do not appear in ON state simultaneously. Otherwise, two power sources may become connected in parallel or shorted to the ground terminal, and the threat of damage may arise.

\[ \frac{di_L}{dt} = - \frac{R_1 + R_4 + R_L + R_C}{L} i_L + \frac{1}{L} V_1 - \frac{1}{L} u_C + \frac{R_C}{L} I_0, \]

\[ \frac{du_C}{dt} = - \frac{1}{C} i_L - \frac{1}{C} I_0. \]  

In a matrix form the system (1) can be presented [8], [10] by

\[
\frac{dx}{dt} = A_1 x + B_1 u, \tag{2}
\]

where \( x = (i_L, u_C)^T \) is the vector state quantities, \( u = (V_1, V_2, I_0)^T \) is the vector of independent quantities, and matrices \( A_1 \) and \( B_1 \) are:

\[
A_1 = \begin{bmatrix}
- \frac{R_{ch1} + R_C}{L} & -1 \\
\frac{1}{C} & 0
\end{bmatrix}, \tag{3}
\]

\[
B_1 = \begin{bmatrix}
\frac{1}{L} & 0 & \frac{R_C}{L} \\
0 & \frac{R_C}{L} & 0
\end{bmatrix}. \tag{4}
\]

where \( R_{ch1} = R_1 + R_A + R_L \). Correspondingly for the circuits 2 and 3 in Fig. 4 the matrices \( A_2, B_2 \) and \( A_3, B_3 \) can be expressed as follows:

\[
A_2 = \begin{bmatrix}
\frac{R_{ch2} + R_C}{L} & -1 \\
\frac{1}{C} & 0
\end{bmatrix}, \tag{5}
\]

\[
B_2 = \begin{bmatrix}
0 & 1 & \frac{R_C}{L} \\
0 & 0 & - \frac{1}{C}
\end{bmatrix}, \tag{6}
\]

where \( R_{ch2} = R_2 + R_s + R_L \), and

\[
A_3 = \begin{bmatrix}
\frac{R_{ch3} + R_C}{L} & -1 \\
\frac{1}{C} & 0
\end{bmatrix}, \tag{7}
\]

\[
B_3 = \begin{bmatrix}
0 & 0 & \frac{R_C}{L} \\
0 & 0 & - \frac{1}{C}
\end{bmatrix}. \tag{8}
\]

where \( R_{ch3} = R_A + R_L \).

![Fig. 2. Structure of the converter.](image)

![Fig. 3. PWM control signals of switches.](image)

### III. MODELING OF STEADY STATE CONDITIONS

#### A. Analytical averaged model of the converter

To obtain the steady state values of the output quantities \( y = (i_1, i_2, V_0)^T \), we will follow the well established methodology of averaging the system of differential equations that describe circuit of the converter dependent on the status of control switches [7]–[9]. The application of the method was already demonstrated for multiple-input [10] and multiphase [11] converters.

Following the Kirchhoff’s laws the systems of differential equations are composed for variables for each circuit shown in Fig. 4. For example, for the first circuit in Fig. 4:

\[
\begin{align*}
\frac{di_1}{dt} & = \frac{-R_1 + R_A + R_L + R_C}{L} i_L + \frac{1}{L} V_1 - \frac{1}{L} u_C + \frac{R_C}{L} I_0, \\
\frac{du_C}{dt} & = \frac{1}{C} i_L - \frac{1}{C} I_0.
\end{align*}
\]

Then the averaged equations system can be written [8]

\[
\frac{dx}{dt} = Ax + Bu, \tag{9}
\]

where averaged matrices are:

\[
A = d_1 A_1 + d_2 A_2 + (1 - d_1 - d_2) A_3, \tag{10}
\]

\[
B = d_1 B_1 + d_2 B_2 + (1 - d_1 - d_2) B_3, \tag{11}
\]

and the duty cycles of PWM control signals of the switches \( S_1 \) and \( S_2 \) are:
\[ d_1 = \frac{T_1}{T_S} \] \hspace{1cm} (12)
\[ d_2 = \frac{T_2}{T_S} \] \hspace{1cm} (13)

In the steady state conditions all derivatives are assumed to be equal to zero, and the solution of (9) is obtained by solving the system of linear equations [8]

\[ \mathbf{x}_{st} = -\mathbf{A}^{-1}\mathbf{Bu}. \] \hspace{1cm} (14)

As described in [8] the steady state solution of the output quantities \( y = (i_1, i_2, V_0)^T \) can be expressed in the following

\[ y_{st} = \mathbf{Cx}_{st} + \mathbf{Eu}, \] \hspace{1cm} (15)

where:

\[
\mathbf{C} = d_1 \mathbf{C}_1 + d_2 \mathbf{C}_2 + (1 - d_1 - d_2) \mathbf{C}_3, \hspace{1cm} (16)
\]
\[
\mathbf{E} = d_1 \mathbf{E}_1 + d_2 \mathbf{E}_2 + (1 - d_1 - d_2) \mathbf{E}_3, \hspace{1cm} (17)
\]

and

\[
\mathbf{C}_1 = \begin{bmatrix} 1, & 0 \\ 0, & 0 \\ R_C, & 1 \end{bmatrix}, \hspace{1cm} (18)
\]
\[
\mathbf{C}_2 = \begin{bmatrix} 0, & 0 \\ 0, & 1 \\ R_C, & 1 \end{bmatrix}, \hspace{1cm} (19)
\]
\[
\mathbf{C}_3 = \begin{bmatrix} 0, & 0 \\ 0, & 0 \\ R_C, & 1 \end{bmatrix}, \hspace{1cm} (20)
\]

\[
\mathbf{E}_1 = \mathbf{E}_2 = \mathbf{E}_3 = \begin{bmatrix} 0, & 0, & 0 \\ 0, & 0, & 0 \\ 0, & 0, & -R_C \end{bmatrix}. \hspace{1cm} (21)
\]

Equation (15) was used to calculate steady state currents \( i_{1st} \) and \( i_{2st} \) of two supply sources, and converter output voltage \( V_{0st} \).

**B. Verification using Spice modeling**

Solution of (15) and its implementation with Matlab were verified by simulating schematics shown in Fig. 5 using LTSPICE simulation software. The period of PWM signal was accepted \( T_S = 10\mu s \).

From the waveforms given in Fig. 6 it was obvious that the converter was operating in the continuous current mode (CCM). The used method of differential equations averaging is applicable for converter modeling in CCM [8]. Steady state output voltage \( V_{0st} \) was estimated at the time moment \( t = 1.5 \text{ ms} \) (see Fig. 6). Comparison of \( V_{0st} \), values obtained using LTSPICE simulation and those calculated from (15) have shown very good agreement at several combinations of duty cycles \( d_1 \) and \( d_2 \), and load current \( I_o \). The mismatch between the values was only at the milivolt level.

\[ V_{0st} = c_3 i_{Lst} + c_2 u_{Cst} + c_1 V_1 + e_2 V_2 + e_1 I_0, \] \hspace{1cm} (22)

where \( c_1, c_2, c_3, e_1, e_2, e_3 \) are corresponding elements of matrix \( \mathbf{C} \), and \( i_{Lst} \) and \( u_{Cst} \) are corresponding elements of matrix \( \mathbf{E} \). Expressions for \( i_{Lst} \) and \( u_{Cst} \) are obtained substituting (7) and (8) to (11). Then from (22) the duty cycle can be expressed

\[ d_2 = \frac{V_{0st} - d_1 V_1 + I_0 (d_1 R_{ch1} + (1 - d_1) R_{ch2})}{V_2 + I_0 (R_{ch3} - R_{ch2})}. \] \hspace{1cm} (23)

The correctness of the expression was verified by numerically solving (15) using Matlab \( \text{fsolve} \) function. No mismatch was observed. The duty cycle \( d_2 \) calculated according to (23) must also be constrained by the following inequality

\[ 0 < d_1 + d_2 \leq 1. \] \hspace{1cm} (24)

If calculated \( d_2 \) does not comply to the (24) that means the converter is not able to ensure the requested output voltage.

**IV. A SUGGESTION FOR THE CONTROL ALGORITHM**

Usually a controller of a switching converter is implemented as a PID controller. It takes feedback from the output voltage (and current) and seeks to adjust duty ratio such that to keep \( V_0 \) equal to reference voltage \( V_{ref} \). In our case \( V_{ref} = V_2 \). Influence of load current and input voltage fluctuations on \( V_0 \) must be suppressed (first criterion) by the controller operating in disturbance rejection mode. In the
analyzed two input converter two control variables are available ($d_1$ and $d_2$). It could be many combinations of $d_1$ and $d_2$ that ensure the preset output voltage. Therefore, we suggest that the second criterion for the adaptive control algorithm should be the minimization of the power $P_2$:

$$P_2 = i_{2a} V_2,$$  \hspace{1cm} (25)

where $i_{2a}$ can be obtained from (15). In Fig. 7(a) the dependences of $P_2$ vs. $d_1$ at two load currents are shown, assuming that $d_2$ is selected using (23) and (24) in order to ensure output voltage equal to $V_2$. It can be seen that there exists an optimal value of $d_1 = d_1^*$ which corresponds to the minimum of $P_2$.

![Fig. 7. The reserve source power: a – the duty cycle $d_2$; b – the output voltage; c – dependence on duty cycle $d_1$.](image)

It must be commented that the drop of the $P_2$ curve in the range of $d_1 > 0.85$, is due to the fact that it is no longer possible to find a value for $d_2$ such that to keep $V_0 = V_2$. It can be seen from Fig. 7(c). Indeed, the $d_2$ was calculated according to (23) but finally was set to $d_2 = 1 - d_1$ if the condition $d_1 + d_2 \leq 1$ was not satisfied. Therefore, the range of $d_1 > 0.85$ (when $I_0 = 5$ A) should not be treated as a normal operating region of the converter.

When plotting Fig. 7 the voltage of the source $V_j$ was modeled by the function presented in [12], which describes generated voltage of a photovoltaic cell (PV) in respect to its load current $i_{PV}$. In particular the output voltage of the PV cell PV2-70W can be expressed [12]

$$V_j = V_{oc} + V_p \ln(1 - i_{PV} / I_{sc}) - R_s i_{PV},$$  \hspace{1cm} (26)

where PV cell parameters in the model are $V_{oc} = 22.2$V, $V_p = 1.06$V, $I_{sc} = 4.27$A, and $R_s = 0.44\Omega$. The current of the PV cell was assumed

$$i_{PV} \approx d_1 I_0.$$  \hspace{1cm} (27)

The following parameters of the converter (Fig. 2) components were used to plot Fig. 7: $V_2 = 12$V, $L = 100\mu$H, $C = 100 \mu$F; $R_L = 0.5\Omega$, $R_C = 0.1\Omega$, $R_1 = R_2 = 0.5\Omega$, $R_{ld} = R_{d} = 0.1\Omega$.

The optimal value of $d_1 = d_1^*$ seems to be close to the value $d_{imp} = I_{imp} / I_0$ corresponding to the maximum power point (MPP) of the PV cell. However, the internal resistances of sources $U_1$ and $U_2$ may have an influence on $d_1^*$, which is yet not investigated.

V. CONCLUSIONS

The results of steady state modeling of the considered two-input DC-DC converter approve its suitability for combining unstable harvested power with reserve source power for DC load supply. The control variables are duty cycles of electronic switches connected in the branches of each power source. By manipulating these duty cycles, the output voltage may be kept constant in the certain range of renewable source power fluctuations and load current demand.

It was demonstrated that there exists an optimal set of control variables (duty cycles) that minimize the power drawn from the reserve source.

REFERENCES


