Abstract—An optimization of parameters of movement of ions of alive tissues and also parameters of movement of ions of different drug implemented in alive tissues can be provided using appropriate parameters of external low frequency electromagnetic field on the basis of mathematical results. The obtained engineering basis for optimization of the process of medical therapy is the main result of the present investigation. A new mathematical model for description and investigation of movement of ions (including their velocity) in alive tissues under influence of external low frequency electromagnetic field is performed in the article.

Index Terms—Alive tissues, linear medium, low frequency electromagnetic signals, physiotherapy, space-temporal signal processing.

I. INTRODUCTION

It is well known that the percent of liquid in alive tissues is the biggest one. Usually the electrolytic dissociation is the state of these liquids. The ions of liquid are in permanent movement, including movement under influence of external electromagnetic field [1]. Usually the increasing of temperature of alive tissues is as result of influence of external high frequency electromagnetic field. When there is an influence of external static electric/magnetic field or low frequency electromagnetic field the result is different without increasing of temperature of alive tissues. In this case the influence of internal low frequency electromagnetic fields is the reason for generation of “internal” electromagnetic signals with interesting 3D configuration as “reaction” of system (alive tissues). Usually the alive tissues are modelled approximately as linear environment [2]–[5]. Some results on investigation on internal signals of alive tissues under influence of external low frequency electromagnetic field are described in this paper. Let us think there are ions along x-axis with electrical charge \( q(x) \). The quantity of electrical charge \( Q \) between two points \( X_1, X_2 \) on axis \( x \) can be calculated by

\[
Q = \int_{X_1}^{X_2} q(x)dx.
\]  (1)

If \( X_1 = 0 \) and there is a movement of point \( X_2 \) on axis \( x \)

\[
Q(t) = \int_0^{X_2(t)} q(x)dx.
\]  (2)

The rate of change of electrical charge in the case of movement of point \( X_2 \) on axis \( x \) is

\[
\frac{dQ(t)}{dt} = \frac{d}{dt} \int_0^{X_2(t)} q(x)dx.
\]  (3)

Let us think that the ions are on an infinite axis \( x \), which crosses plane \( \alpha \) at point \( \alpha \) and that the calculation of electrical charge \( Q \) starts when the first ion goes through point \( \alpha \). In this case the modulus of vector of density \( |\vec{\delta}(t)| \) at point \( \alpha \) of electrical current \( i(t) \) can be calculated by the following

\[
|\vec{\delta}(t)| = \frac{dQ(t)}{dt} = \frac{d}{dt} \int_0^{X_2(t)} q(x)dx.
\]  (4)

The mass and electrical charge of different ions are different. In the case of movement of one kind of ions

\[
q(x) = q = \text{const}.
\]  (5)

It is well known that the percent of ions of \( Na^+ \) or \( Cl^- \) is the biggest in liquids of alive tissues. Because of that condition (5) can be accepted as approximately correct. Therefore often in practice the modulus of vector of density \( |\vec{\delta}(t)| \) at point \( \alpha \) of electrical current \( i(t) \) can be calculated by
The electrical current $i(t)$ through surface $S$ as part of plane $\alpha$, can be calculated by

$$i(t) = \int_{S(t)} \frac{dX(t)}{dt} q(x) dx ds,$$

where $q(x)$ represents surface charge density ($\text{Cb/m}^2$).

The spectral function $S_i(\omega)$ of current $i(t)$ can be calculated by

$$\hat{S}_i(\omega) = q^2 \int_0^\infty \frac{dX(t)}{dt} e^{-j\omega t} dt,$$

where the surface current density was assumed here constant on surface $S$, and where $\omega$ is the angular frequency.

It is clear that there is first of all a transformation of energy of external static or low frequency electromagnetic field to energy of movement of ions in live tissues, but not as thermal energy. The parameters of this movement should depend on the way of application of external electromagnetic field. The movement of ions is the reason for existing of “internal” signals in alive tissues as reaction of influence of external signals. Therefore the alive tissues can be discussed as one system with external influence and internal reaction. This is a new method for investigation of influence of external low frequency electromagnetic field on alive tissues. The parameters of “internal” signals depend on the velocity $V(t)$ of movement of ions according to (6).

Therefore as first step in investigation on internal signals in alive tissues under influence of external low frequency electromagnetic field should be the determination of components of velocity $V(t)$ of movement of ions in 3D. The above mentioned new method and the obtained results would be important for optimization of processes in medical therapy.

II. ANALYSIS OF SPACE-TEMPORAL PROCESSING IN THE CASE OF INFLUENCE OF LOW FREQUENCY ELECTRIC AND MAGNETIC FIELD ON IONS IN LINEAR MEDIUM

The presented investigation is on space-temporal processing of simultaneous influence of two independent low frequency or static magnetic and electrical fields on ions in linear medium.

A. Mathematical Model

The mutual disposition of both independent vectors of electrical intensity $\vec{E}(x, y, z, t)$ and magnetic induction $\vec{B}(x, y, z, t)$ in the case of simultaneous influence of low frequency electrical and magnetic field [4], [5] on the ions can be seen at Fig. 1.

The origin of coordinate system is at the point at which the charged particle is situated. This point can be any point in space, in which both signals act. The vector of the magnetic induction $\vec{B}(x, y, z, t)$ coincides with axis $Z$. The angle between vector of electrical intensity $\vec{E}(x, y, z, t)$ and axis $Z$ is $\gamma$. The angle between projection of this vector on the plane XOY and axis $X$ is $\beta$. The components of vector of electric intensity $\vec{E}(x, y, z, t)$ on the axes $X, Y, Z$ of coordinate system are $E_x, E_y, E_z$. The differential equation (9) of movement of ion can be obtained as following

$$m \frac{d^2 \vec{r}(t)}{dt^2} = q \vec{E}(x, y, z, t) + q \frac{d\vec{r}(t)}{dt} \times \vec{B}(x, y, z, t),$$

where $m$ is the mass of ion, $q$ is the electric charge of ion, $\vec{r}$ is tangential trajectory vector of the movement of the ion.

The components $E_x, E_y, E_z$ of the vector of electric intensity $\vec{E}(x, y, z, t)$ can be obtained as following, by taking into account Fig. 1:

$$E_x = |\vec{E}(x, y, z, t)| \sin \gamma \cos \beta,$$

$$E_y = |\vec{E}(x, y, z, t)| \sin \gamma \sin \beta,$$

$$E_z = |\vec{E}(x, y, z, t)| \cos \gamma.$$
\[
m \frac{d^2 z(t)}{dt^2} = qE(x, y, z, t)\cos \gamma.
\] (15)

B. Experimental Investigations and Discussion

It is known that sodium ions \( \text{Na}^+ \) have a considerable percentage in the composition of living tissue. Therefore, the movements of sodium ions \( \text{Na}^+ \) under influence of “external” signals with certain parameters are described in this article as examples. Some results of visualization of movement of ions, which have been obtained by computer’s methods for solution of the system of differential equations (13)–(15) using MATLAB [6]–[9] have been performed as well. The used input data are often used in medical applications.

C. Influence Only of Low Frequency Magnetic Signal on the Ions

The parameters of magnetic signal are described by

\[
E(x, y, z, t) = 0 \land \dot{B}(x, y, z, t) \neq 0.
\] (16)

Modification (17)–(19) of the system (13)–(15) of differential equations can be obtained by taking into account (16). The investigations have been performed for permanent static magnetic field with modulus of magnetic induction \( \dot{B} = 1mT \). The trajectory of movement of sodium ions \( \text{Na}^+ \) can be seen on Fig. 2:

\[
m \frac{d^2 x(t)}{dt^2} = qB(x, y, z, t) \frac{dy(t)}{dt},
\] (17)

\[
m \frac{d^2 y(t)}{dt^2} = qB(x, y, z, t) \frac{dx(t)}{dt},
\] (18)

\[
m \frac{d^2 z(t)}{dt^2} = 0.
\] (19)

![Fig. 2. The trajectory of movement of sodium ions \( \text{Na}^+ \) in the case of influence of magnetic field only.](image)

The sodium ions \( \text{Na}^+ \) enter in the magnetic field with a determined starting speed, moving in a straight line along z axis. It is seen that under the action of the magnetic field the trajectory of movement of ions is hereby of the type of spiral with radius, which is gradually increased. It is clear that the magnetic field gives an additional rotation of the ions with the continuous increase of the radius of rotation.

D. Simultaneous Influence on the Ions of Both Simple Sinusoidal Magnetic Signal and Permanent Electric Field (Process of Magneto-Therapy with Ionoforesis)[10]

The parameters of “external” electric and magnetic signals are described by

\[
\dot{E}(x, y, z, t) = \text{const} \land \dot{B}(t) = \dot{B}_m \cos \omega_3 t \land
\]

\[
\dot{B}(x, y, z) = \text{const} \land \omega_2 = \text{const} \land \beta = 45^\circ \land \gamma = 45^\circ,
\] (20)

where \( \dot{B}_m \) is the amplitude of magnetic induction at all points of homogenous sinusoidal magnetic field, \( \omega_3 \) is the frequency of the sinusoidal magnetic signal.

The system of differential equations (21)–(22) is a modification of system differential equations (13)–(15) taking in account (20). The solutions of the system of differential equations (21)–(22) can be seen on Fig. 3. The dimensions for the trajectory of movement of ions are \([m.10^{-1}]\) and the dimension for the velocity of ions is \([m.s^{-1}]\).

\[
m \frac{d^2 x(t)}{dt^2} = q[E \sin \gamma \cos \beta + \frac{dy(t)}{dt} B_m \cos \omega_3 t],
\] (21)

\[
m \frac{d^2 y(t)}{dt^2} = q[E \sin \gamma \sin \beta + \frac{dx(t)}{dt} B_m \cos \omega_3 t],
\] (22)

\[
m \frac{d^2 z(t)}{dt^2} = qE \cos \gamma.
\] (23)

The values of parameters in differential equations system (21)–(23) are \( E = 100[V/m], \) \( \dot{B}_m = 30[mT], \) \( \omega_3 = 2\pi \times 50[1/s] \).

The components of velocity and trajectory of movement of ions on the axes \( X, Y, Z \) can be seen at Fig. 3(a), Fig. 3(b) and Fig. 3(c). The trajectory of movement of ions can be seen at Fig. 3(d).

If the trajectories of movement of ions from Fig. 2 and Fig. 3(d) would be compared, it would be obtained an important conclusion: it is enough to add even a permanent static electric field (during the process of magneto-therapy) at a low frequency harmonic magnetic signal for providing new opportunities for “management” of trajectory of movement of ions in living tissues. This is a prerequisite to achieve better and faster therapeutic effect. Therefore the simultaneous application of “external” low frequency electric and magnetic signals can be a new method in modern physiotherapy. Therefore the study of the movement of ions in living tissue just under the influence of the magnetic field is no longer enough. This research and visualization of the movement of ions in the tissues should be made under the simultaneous influence of both “external” electric and magnetic signals, which is the purpose of this article.
On the basis of discussion and in relation to (6), the next important conclusion can be obtained: the forms of the components of the “internal” signals (reaction of the system) along the three axes X, Y and Z are the same as the shape of the components of the velocity of the ions along the respective axes. Therefore, in fact the solution of the differential equations (13)–(15) defines the reaction of system (living tissue).

III. CONCLUSIONS

1. A new engineering method for investigation of influence of external low frequency electromagnetic signals on alive tissues is described in the article. The external electromagnetic signals are described as influence on one linear system (alive tissues) and the reaction of this system is described through “internal” low frequency electromagnetic signals. According to (1)–(7) the reaction (“internal” low frequency electromagnetic signals) of system (alive tissues) depends on the velocity of movement of ions under influence of external low frequency electromagnetic field.

2. A new mathematical model for description and investigation of movement of ions (including their velocity) in alive tissues under influence of external low frequency electromagnetic field is performed in the article (Fig.1 and (9)–(11)).

3. The obtained results (Fig. 3) for components of velocity of movement of ions in 3D can be put as functions in (1) and (3) for investigations of “internal” signals (reaction of system).

4. An optimization of parameters of movement of ions of alive tissues and also parameters of movement of ions of different drug implemented in alive tissues can be provided using appropriate parameters of external low frequency electromagnetic field on the basis of mathematical results.

REFERENCES