Design of the Deadbeat Controller with Limited Output

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Introduction

Deadbeat control system is digital control system. The deadbeat control could be used in systems where the known finite settling time is required. In this article, the presented deadbeat controller design is based on the object z-transfer function. The overall structure of the control system is shown in Fig. 1.

![Fig. 1. Structure of the control system: g – reference input; u – manipulated variable (controller output); y – system output](image)

It is known that the deadbeat controller transfer function $W_{DC}(z)$ may be written as the following [1]

$$W_{DC}(z) = \frac{Q(z)}{1 - P(z)},$$

where $Q(z)$, $P(z)$ are the polynomials of the transfer function of the deadbeat controller. The coefficients of the polynomials $Q(z)$, $P(z)$ are found by using these equations:

$$q_0 = \frac{1}{b_1 + b_2 + \ldots + b_m},$$
$$q_i = q_0 \cdot a_i,$$
$$p_i = q_0 \cdot b_i,$$  \quad i = 1, \ldots, m,

where $a_i$, $b_i$ are coefficients of the polynomials of the continuous object’s z-transfer function $W_{CO}(z) = \frac{B(z)}{A(z)}$; $m$ – order of the object transfer function. The coefficients of polynomial $Q(z)$ can be expressed as [1]:

$$q_0 = u(0),$$
$$q_i = u(i) - u(i-1), \quad i = 1, \ldots, m.$$  \quad (3)

Thus the initial value of manipulated variable $u(0)$ depends only on the sum (2) of the object’s z-transfer function coefficients $b_i$.

The main drawback of the deadbeat controller is that the sampling time $T_0$ and manipulated variable $u(0)$ are inversely proportional. $T_0$ is conditioned by $u_{\text{max}}$, which depends from various factors.

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By multiplying the numerator and the denominator polynomials of the controller’s transfer function (1) by an additional polynomial $C(z)$ [1] we get

$$W_{DC}(z) = \frac{Q(z)C(z)}{1 - P(z)C(z)},$$

where $C(z) = 1 + c_1 \cdot z^{-1}$. The coefficients of the deadbeat controller are found using these equations:

$$q_0 = \frac{1}{(b_1 + b_2 + \ldots + b_m)(1 + c_1)},$$
$$q_1 = q_0 \cdot (a_1 + a_{i=1..m} \cdot c_1),$$
$$p_1 = q_0 \cdot (b_1 + b_{i=1..m} \cdot c_1), \quad i = 1, \ldots, m.$$  \quad (5)

The deadbeat controller’s properties depend from coefficient $c_1$. If we assume that $u(0) \leq u_{\text{max}}$, then using equations (3) and (5), the following can be written

$$c_{10} = c_1 = \frac{1}{U_{\text{max}} \cdot (b_1 + b_2 + \ldots + b_m)},$$

where $c_{10}$ – a coefficient limiting the value of the manipulated variable from the top at time moment zero. So, if $c_{10} \leq c_{i10}$ then $u(0) \leq u_{\text{max}}$.

Let us assume that $-u(0) \leq u(1) \leq u(0)$, then express $u(0)$ and $u(1)$ in terms of $q_i$ by using (3) and (5)

$$-q_0 \leq q_0 + q_0a_1 + q_0c_1 \leq q_0.$$  \quad (7)

Dividing inequality (7) from $q_0$ ($q_0 > 0$) we get

$$-2 \leq a_1 + c_1 \leq 0.$$  \quad (8)

Significant (8) boundary is expression
\( c_{11} = c_1 = -a_1 \), \( \text{(9)} \)

where \( c_{ij} \) – a coefficient limiting the value of the manipulated variable at time moment one. So, if \( c_{ij} \leq c_{11} \) then \( |u(1)| \leq u_{\text{max}} \).

Let us assume that \( -u(0) \leq u(2) \leq u(0) \), then express \( u(0) \) and \( u(2) \) in terms of \( q_i \) by using (3) and (5)

\[
- q_0 \leq q_0 a_1 + q_0 c_1 + q_0 a_2 + q_0 a_1 c_1 \leq q_0.
\]

Dividing the inequality (10) by \( q_0 \) \((q_0 > 0)\) and rearranging we get

\[
-2 - a_1 - a_2 \leq c_1 (1 + a_1) \leq -a_1 - a_2.
\]

The expression \((1+a_1)\) in inequality (11) may be either positive or negative. If it is positive, then

\[
\frac{-2 - a_1 - a_2}{1 + a_1} \leq c_1 \leq \frac{-a_1 - a_2}{1 + a_1} \Rightarrow c_{121}.
\]

If \((1+a_1)\) is negative, then

\[
\frac{-2 - a_1 - a_2}{1 + a_1} \geq c_1 \geq \frac{-a_1 - a_2}{1 + a_1} \Rightarrow c_{122}.
\]

If \(|a_1| > |a_2|\) in equation (11), then:

\[
c_{121} = \frac{-2 - a_1 - a_2}{1 + a_1},
\]

\[
c_{122} = \frac{-a_1 - a_2}{1 + a_1},
\]

where \( c_{121}, c_{122} \) - coefficients limiting the value of the manipulated variable at time moment two.

Such procedures are included in the design of deadbeat controller with limited manipulated variable.

Using equations (6), (9), (14) and (15) it is necessary to find the values of coefficients \( c_{ij} \), \( c_{11}, c_{121} \), \( c_{122} \) at different sampling time \( T_0 \) values and to draw dependencies \( c_{10} = f(T_0), c_{11} = f(T_0), c_{121} = f(T_0), c_{122} = f(T_0) \) (see Fig.2.).

The non-highlighted area is the area from which it is possible to choose a value for \( c_1 \) and then, by using it, define a suitable sampling time \( T_0 \), \( c_1 \) value may be chosen from the leftmost point of the non-highlighted area (see Fig.2.), the manipulated variable \( u \) will be limited by this inequality: \( u_{\text{max}} = |u(0)| = |u(1)| > |u(2)| \). After the value of \( c_1 \) is chosen it is possible to define a suitable sampling time \( T_0 \). Once this is done, the coefficients of the deadbeat controller are found using (5) and the design procedure is over.

**Simulation and experiments of the deadbeat control system**

Performance of the a forementioned controller design procedure can be found through the simulations and experiments.

A third order continuous object \([2]\) was chosen and its structure is shown in Fig. 3.

If we consider the parameters of a chosen object indicated in Fig. 3, we can write continuous transfer function

\[
W(s) = \frac{1}{(5s + 1) \cdot (s + 1) \cdot (5s + 1)}.
\]

The object response \( y \), simulated by Matlab software, to the unit step input \( u \), is depicted in Fig. 4.
Fig. 4. Object response (2-\(y\)), simulated by Matlab software, to the unit step input (1-\(u\))

Continuous object response \(y\) to the unit step voltage input \(u\), is shown in Fig. 5.

By comparing the real object response with the one simulated by Matlab, it can be seen that they are fairly similar. The real object response is with a disturbance. The transfer function given in the expression (16) will henceforth be used to design the deadbeat controller.

The design of the deadbeat controller is executed in Matlab [3]. The continuous object transfer function is converted to discrete time assuming a zero order hold on the inputs. By using equations (1) and (2), while holding \(u(0)=3.0\), we get the transfer function of the deadbeat controller

\[
W_{DC}(z) = \frac{3.0007 - 2.553 z^{-1} + 0.559 z^{-2} - 0.0068 z^{-3}}{1 - 0.4525 z^{-1} - 0.5238 z^{-2} - 0.0237 z^{-3}}, \quad T_0 = 4.35s.
\] (17)

By using a method, which includes Matlab simulation and experimenting with the PLC, we get the deadbeat control system (see Fig. 1.) responses \(y\) to the unit step input reference signal \(g\).

Fig. 6. shows that the system response has a finite settling time. The response ends in 8 s after four steps of the control signal. The system response does not have any indications of overshooting.

The next step will be design of the deadbeat controller with limited output using equations (1) and (5). By using the additional procedure shown in Fig. 2, for choosing coefficient \(c_1\), we find the transfer function of the deadbeat controller, holding that \(u(0)=3.0=\hat{u}(1)\), because \(c_1^{10}=\hat{c}_1=1.2826\).

\[
W_{DC}(z) = \frac{3.0 + 0.0091 z^{-1} - 3.5635 z^{-2} + 1.6735 z^{-3} - 0.333 z^{-4}}{1 - 1.1466 z^{-1} - 0.4542 z^{-2} - 0.3668 z^{-3} - 0.0324 z^{-4}}, \quad T_0 = 2.539s.
\] (18)

By using a method, which includes Matlab simulation, we get the deadbeat control system response \(y\) to the unit step input reference signal \(g\).

Fig. 8. shows shows that the system response has a finite settling time. The response ends in 8 s after four steps of the control signal. The system response does not have any indications of overshooting. The value of the control signal \(u(2)\) is negative (then \(g=1\) and thus cannot be realized with PLC analogue output module.)
Fig. 8. Matlab simulated deadbeat system with limited controller output response (2–y) to unit step reference input signal (1–g), 3 – controller output u

Fig. 9. PLC implemented deadbeat system with limited controller output response (2–y) to step reference input signal (1–g), 3 – controller output u

So experiment with the PLC includes investigation of the deadbeat system response to step reference signal, when system output is \( y=1 \).

Fig. 9. shows that the system response has a overshoot of 7%, while the transition takes more than 6 sampling time units.

It can be concluded that the responses of the experiments follows the system’s response to the Matlab simulation with an error.

Conclusions

The presented procedure for choosing the deadbeat controller parameter \( c_1 \) allows for a decrease in the sampling time \( T_0 \), without increasing the maximum allowed value of the control signal.

The following observations were made based on such results. Simulation results show that even though the control increases by one-step, the length of the settling time of the system response can be lower than that of the deadbeat controller without any modifications.

It was found that the deadbeat controller is not of high quality when taking into account the changes in the parameters of an object. The quality of the deadbeat control system is influenced by the accuracy of the transfer function of the object. That is why it is crucial to have a reliable method for recognizing the transfer function of the object.

References


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There is presented a method for finding the parameters of the deadbeat controller in Matlab environment. The method is based on the introduction of an additional polynomial into the transfer function of the controller. The method for determining the additional polynomial coefficient of a deadbeat controller is based on creating the family of the coefficient curves and defining the permissible selection area. The method was tested by using simulations in Matlab environment and realizing the deadbeat control system for the third order object in the PLC. Simulation results in Matlab show that even though the control increases by one-step, the settling time of the system response can be lower than that of the deadbeat controller without any modifications. Based on the obtained results it can be concluded that the results confirm the idea of defining the parameters of the transfer function of a deadbeat controller with a limited output. Ill. 9, bibl. 3 (in English; abstracts in English and Lithuanian).


Pateiktas aperiodinio reguliatoriaus parametrų nustatymo Matlab aplinkoje metodas, pagrįstas papildomo polinomo įvedimu į reguliatoriaus perdavimo funkciją. Aperiodinio reguliatoriaus papildomo polinomo koeficiento nustatymo metodas remiasi koeficiento kreivių šeimos sudarymu ir leistinos parinkti zonos apibrėžimu. Metodas išbandytas modeliuojant Matlab aplinkoje ir taikant aperiodinio valdymo sistemą trečios eilės objektiui programuojamoje loginiame valdiklyje. Modeliavimo Matlab rezultatai rodo, kad nors valdymas pailgėja vienu žingsniu, tačiau sistemos reakcijos pereinamojo proceso trukmė gali būti trumpesnė negu aperiodinio nemodifikuoto reguliatoriaus. Remiantis gautais rezultatais galima teigti, jog rezultatų patvirtina aperiodinio reguliatoriaus su apribotu išėjimu perdavimo funkcijos parametrų nustatymo idėją. Il. 9, bibl. 3 (anglų kalba; santraukos anglų ir lietuvių k.).