Introduction

The generalized structure of the symmetrically coupled microstrip multiconductor line (MMCL) is shown in Fig. 1. The MMCL consists of a dielectric substrate with a conducting layer (reference or grounded conductor) on one side and microstrip conductors on the other side. The number of the microstrip conductors, their width and space between them can be specified independently. Prominent feature of symmetrically coupled MMCLs is their “mirror” structure, i.e. equality of width of the microstrip conductors $W_i = W_{N-i+1}$ located on identical distances concerning a vertical plane of symmetry of the MMCL. Due to their planar design, these lines are broadly used in many microwave application areas [1–3]. MMCL are also successfully used as physical models for designing various microwave devices [4–13].

![Fig. 1. Generalized structure of the symmetrically coupled microstrip multiconductor line: 1 – reference (grounded) conductor; 2 – dielectric substrate; 3 – microstrip conductors; 4 – plane of symmetry](image)

It is known, that, generally, in a lossless MMCL consisting of $(N - 1)$ conductors can propagate $N$ normal modes (or normal waves), which are characterized by $N$ propagation factors $\beta_i$ and $N^2$ characteristic impedances $Z_{ii}$ (where $k$ is type of normal mode; $i$ is conductor number) [14].

Phase and group velocities of different types of normal waves differ from each other in the MMCL, as a result the interference of the propagated normal waves occurs in the MMCL and transmitted signals suffer from distortions. Therefore microwave devices, in which models the MMCL are used, should be designed so that in them one normal wave of the specified type could propagate only [15].

Synthesis of the device is a procedure of finding of its constructive dimensions and parameters according to the given electric characteristics of this device. Synthesis problems are of particular interest for creating CAD systems for microwave devices, and many researches were intensively solving the problems in the past decade [16]–[19]. E.g. Rawat and Ghannouchi [16] have derived closed-form expressions which relate electrical characteristics of the designed devices and its constructive parameters. Nomograms and tables, relating electrical characteristics of the synthesized device and its constructive parameters were created by Chiang et al. [17] using a commercial simulator. In case of the MMCL operating in normal mode, synthesis is an iterative process of finding of MMCL topological sizes and the constructive parameters, which guarantee excitation and propagation of the single normal wave in the line at the applying specified modal voltage. Han et al. [3] have derived closed-form expressions linking up electrical characteristics of the four-conductor coupled microstrip line with its constructive parameters. Values of the constructive parameters here are obtained from the commercial simulator. Yioultis et al. [18] have analyzed MMCL using the finite difference method. In order to reduce computational time they have analyzed discrete models of MMCL with optimized discretization step. Lee, and Tsai [19] have got closed-form expressions and nomograms associating parameters of equivalent circuits of the three-conductor MMCL with electrical characteristics of the MMCL.

A survey of literature reveals that there is no well-established systematic design technique for synthesis of an MMCL, operating in the normal mode. In the absence of such a technique, circuits are typically designed using extensive software simulations where high-frequency simulators (mostly commercial) are used for choosing the required design parameters for a good coverage [16–18].
In this paper we propose an original synthesis technique of six-conductor symmetrically coupled MMCL, operating in normal mode. This technique demonstrates rather fast computational efficiency and good accuracy.

**Synthesis Algorithm**

The proposed synthesis algorithm of a symmetrically coupled MMCL is based on an iteration of calculation of modal voltages, and changing structure of the MMCL in order to find such structure of an MMCL, for which amplitudes of modal voltages are equal for every microstrip conductor.

Modal voltages of the line are calculated by finding eigenvalues of the relative effective dielectric permittivity vector solving an eigenvalue equation

$$[C_1][V] = [C^{(i)}][V][\varepsilon_{\text{eff}}],$$

where $[C_1]$ is the matrix of partial capacitances of the MMCL with a certain dielectric permittivity of the substrate; $[C^{(i)}]$ is a similar matrix of the analogical MMCL with free space in place of the dielectric substrate; $[\varepsilon_{\text{eff}}]$ is the vector of relative effective permittivity of MMCL; $[V]$ is the matrix consisting of modal voltage vectors. The analysis of the MMCL can be carried out using various methods, e.g. finite difference [20], partial areas [21] etc. We propose to use the combination of the method of moments and partial images technique [9, 22, 23] for accurate calculation of partial capacitance matrices $[C_1]$ and $[C^{(i)}]$. In general case, modal voltages of a six-conductor microstrip line can be expressed as functions of widths of the conductors

$$V_i = f_i(W_{1&6}, W_{2&5}, W_{3&4}),$$

where $i = 1...6$ is conductor’s number.

It is known from our earlier calculations [24, 25], that in order to obtain normal modes when voltages of the same amplitude are applied to the conductors, lines must be symmetric, i.e. $W_1 = W_6 = W_{1&6}$, $W_2 = W_5 = W_{2&5}$ and $W_3 = W_4 = W_{3&4}$. Furthermore, modal voltages of symmetric conductors are also symmetric, i.e. $V_1 = V_6$, $V_2 = V_5$ and $V_3 = V_4$. Therefore Eq. (2) can be expressed

$$V_i = f_i(W_{1&6}, W_{2&5}, W_{3&4}).$$

The aim of the synthesis is to find such $W_{1&6}$, $W_{2&5}$ and $W_{3&4}$ values, so that modal voltages of the conductors would be differ by no more than the desired error $\Delta V_{\text{max}}$. It was chosen to leave $W_{3&4}$ value fixed, and to change values of $W_{1&6}$ and $W_{2&5}$ in order to obtain the desired modal voltages. In this case, outer conductors (#1 and #6) were wider than inner ones (#3 and #4) for even mode, and narrower for odd mode. It is also convenient to normalize voltages by the voltage of one of the conductors.

control of the difference between voltages. Voltage of any of the conductors could be chosen, and for the sake of simplicity, it was chosen to normalize all voltages by $V_1$.

Considering the above, in order to find the needed widths of the conductors, the following inequality system have to be solved:

$$\left| V_1 - V_j \right| = f(W_{1&6}, W_{2&5}) < \Delta V_{\text{max}},$$

$$\left| V_1 - V_j \right| = f(W_{1&6}, W_{2&5}) < \Delta V_{\text{max}}.$$  \hspace{1cm} (4)

It has been also noticed, that the modal voltage of the conductor strongly depends on its own width, and the dependence on other conductor’s width is quite weak [24, 25], e.g. $V_1$ strongly depends on $W_{1&6}$, but the dependence on $W_{2&5}$ is significantly weaker. The same applies to $V_2$ dependence on $W_{2&5}$ and $W_{1&6}$ . Therefore the above inequality system (4) can be modified to:

$$\left| V_1 - V_j \right| = f(W_{1&6}) < \Delta V_{\text{max}},$$

$$\left| V_1 - V_j \right| = f(W_{2&5}) < \Delta V_{\text{max}}.$$  \hspace{1cm} (4a)

Any numerical technique can be used to find $W_1$ and $W_2$ by alternating between the inequalities, until both inequalities are satisfied.

The secant method [26] was chosen by authors because of its suitability for numerical techniques and good convergence. Recurrence relation for the secant method is given by

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f(x_{n-1}) - f(x_{n-2})},$$

where $x_n$ in our case is the “newest” calculated value of $W_i$; $x_{n-1}$ and $x_{n-2}$ are the values of $W_i$ calculated accordingly on the previous and pre-previous calculation cycles; $f(x_{n-1})$ and $f(x_{n-2})$ are the differences of modal voltages (4a) calculated accordingly on the previous and pre-previous calculation cycles.

Flowchart of the synthesis algorithm of six-conductor symmetrically coupled MMCL, operating in normal modes, is presented in Fig. 2. It consists of 10 steps:

1. Initial values needed for calculation are set: thickness of the dielectric substrate $h$, the relative permittivity of dielectric substrate $\varepsilon_r$, initial conductor widths $W_{1&6}$, $W_{2&5}$ and $W_{3&4}$, gap size between the conductors (in our case) $S_1 = S_2 = ... = S_5 = S$, and the greatest desired voltage error $\Delta V_{\text{max}}$.
2. Modal voltages are calculated according to Eq. (1).
3. It is checked, if the difference between $V_1$ and $V_i$ is greater than $\Delta V_{\text{max}}$, if true, then continue to block 4, if false, continue to block 5.
4. New $W_{1&6}$ is calculated according to Eq. (5).
5. It is checked, if the difference between $V_1$ and $V_i$ is greater than $\Delta V_{\text{max}}$, if true, then continue to block 6, if false, continue to block 8.
6. New $W_{2&5}$ is calculated according to Eq. (5).
7. Modal voltages are calculated according to Eq. (1).
It is checked, if the difference between $V_1$ and $V_3$ is greater than $\Delta V_{max}$, if true, then continue to block 4, if false, continue to block 9.

9. The effective permittivity $\varepsilon_{eff}$ and the characteristic impedance $Z_i$ are calculated.

10. Data is output, calculation ends.

Investigation of the Proposed Techniques

The proposed technique for synthesis of six-conductor symmetrically coupled MMCL, operating in normal mode, was investigated in the following order. Firstly, accuracy of the mathematical model (the analysis stage) used in the proposed technique was tested. Next, the dependence of the width of the conductors $W_i$ and the characteristic impedance $Z_i$ on constructive parameters of an MMCL: space between the conductors $S$, and the permittivity of the substrate $\varepsilon_r$ was investigated.

The search of publications concerning six-conductor MMCL was unsuccessful. Papers presenting investigation of three-conductor [25, 27] and four-conductor MMCL [3, 14, 24] only were found. Therefore, checking accuracy of the mathematical model used in the proposed technique, authors have been compelled to be limited to examples of the above named three- and four-conductor MMCL. In the aforementioned works three- [27] and four-conductor [14] asymmetrically coupled MMCL operating in normal mode have been investigated. The electrical parameters: $V_i$, $\varepsilon_{eff}$ and $Z_i$ were calculated using Eq. (1) by analyzing three- and four-conductor line, according to set parameters: $W_i$, $S$, $h$ and $\varepsilon_r$. Results of the calculations of parameters of the investigated lines are submitted correspondently in Table 1 and Table 2. It is obvious, that the values presented in both Tables in most cases agree within 3 %. Some discrepancy between our results and the results of [14] and [27] can be explained by different investigation techniques. The spectral domain method was used in [14] and [27]. It is possible to tell about good reliability of our obtained results and our proposed technique.

| Table 1. Comparison of parameters of three asymmetrically coupled microstrip lines* obtained using the proposed technique (MoM), and in [27]. |
|---|---|---|---|---|
| Conductor | Mode A | Mode B | Mode C |
| Modal Voltages, V |
| #1 | 1 | 1 | 1 | 1 | 1 | 1 |
| #2 | –0.9 | –0.875 | 0.61 | 0.6 | 1.16 | 1.15 |
| #3 | 0.17 | 0.175 | –0.66 | –0.66 | 1.19 | 1.13 |
| Relative Effective Dielectric Permittivity |
| – | 5.51 | 5.55 | 6.1 | 6.15 | 7.58 | 7.6 |
| Characteristic Impedance, $\Omega$ |
| #1 | 45 | 46.5 | 73.6 | 76 | 106 | 107 |
| #2 | 32.6 | 33 | 55.7 | 57 | 72.6 | 73.5 |
| #3 | 20 | 19.5 | 29 | 30 | 39.6 | 40 |

* $W_1 = 0.3$ mm, $W_2 = 0.6$ mm, $W_3 = 1.2$ mm, $S_1 = 0.2$ mm, $S_2 = 0.4$ mm, $h = 0.63$ mm, $\varepsilon_r = 9.8$.

| Table 2. Comparison of parameters of four asymmetrically coupled microstrip lines** obtained using the proposed technique (MoM), and in [14]. |
|---|---|---|---|---|
| Conductor | Mode A | Mode B | Mode C | Mode D |
| Modal Voltages, V |
| #1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| #2 | 1.14 | 1.17 | 0.33 | 0.33 | –1.61 | –1.64 | –5.1 | –5.2 |
| #3 | 1.14 | 1.17 | –0.33 | –0.33 | –1.61 | –1.64 | 5.1 | 5.2 |
| #4 | 1 | 1 | –1 | –1 | 1 | 1 | –1 | –1 |
| Relative Effective Dielectric Permittivity |
| – | 7.54 | 7.55 | 6.12 | 6.15 | 5.58 | 5.6 | 5.42 | 5.45 |
| Characteristic Impedance, $\Omega$ |
| #1, #4 | 66 | 66 | 49 | 49.5 | 34 | 34.5 | 24 | 24 |
| #2, #3 | 121 | 123 | 83 | 85.5 | 63 | 64.5 | 41 | 42 |

** $W_1 = W_4 = 0.6$ mm, $W_2 = W_3 = 0.3$ mm, $S_1 = S_3 = 0.3$ mm, $S_2 = 0.2$ mm, $h = 0.635$ mm, $\varepsilon_r = 9.8$. 

Fig. 2. Six-conductor symmetrically coupled microstrip line synthesis algorithm.
Further, the proposed technique was used to synthesize six-conductor MMCL, operating in even and odd normal modes, in order to investigate MMCL characteristics dependence on its constructive parameters. These modes were selected due to their potential practical utility, applying to the conductors correspondently equal voltages +1 V, ... , +1 V or applying counter-phase voltages +1 V, −1 V, ... , +1 V, −1 V.

The widths of conductors, which ensured propagation of single normal wave, were searched during the synthesis procedure, accordingly to the defined $S/h$ ratio, permittivity of the dielectric substrate $\varepsilon_r$, and the selected type of normal mode. After finding such widths, the process of synthesis was finished, and the characteristic impedance $Z_{ee,oo}$ of the conductors of the MMCL and relative effective permittivity $\varepsilon_{ref,ef}$ were calculated. The results of the synthesis of the MMCL, operating in even and odd normal modes are shown in Figs 3–7.

Firstly, by examining these figures, it is necessary to note that all diagrams, submitted in Fig. 3, Fig. 4a, and Fig. 5, characterize the MMCL, operating in the even normal mode, and there are inverse relationships in comparison with corresponding diagrams in Fig. 6, Fig. 4b and 7, that characterize the MMCL, operating in the odd normal mode. By further, analyzing the curves presented on the Figs 3 and 6, it can be assumed that a symmetrically coupled MMCL, operating in a normal mode, is a regular multiconductor line in its middle part, i.e. only widths of the external conductors (in our case 1&6) should be changed for rough adjustment of an MMCL.

In order to ensure the even normal wave propagation in an MMCL, the width of the external conductors (in case under consideration these are conductors #1 and #6) should be greater than that of the internal conductors (see Fig. 3), and on the contrary, in case of odd normal wave (see Fig. 6) – the external conductors should be narrower than internal ones ($W_{1,6}/W_{2,5}$). It is necessary to note that at small distances between the conductors (when $S/h < 2$), the ratio of widths of wide and narrow conductors $W/W_{1,6}$ for the even normal wave exceeds the same ratio ($W/W_{1,6}$) more than twice for the odd normal wave (see Fig. 3 and Fig. 6). By analyzing the curves presented in Fig. 3 for the even mode, and Fig. 6 for the odd mode, it is seen that the widths of the external conductors have the most influence on the propagation of
normal waves in an MMCL, while widths of internal conductors vary negligibly while changing $S/h$.

Diagrams presented in Fig. 4 show that the relative effective permittivity $\varepsilon_{eff}/\varepsilon_r$ changes slightly, while varying the space between conductors. For example, at tenfold change of the space between the conductors, the relative effective permittivity $\varepsilon_{eff}$ of the MMCL changes, depending on the permittivity of the dielectric substrate $\varepsilon_r$, within $20$–$35\%$, and in the MMCL, operating in the odd normal mode, under the same conditions, the change is even less: $10$–$17\%$.

Increasing the space between adjacent conductors of the MMCL, causes their characteristic impedance to become comparable and the difference between MMCL, operating in opposite normal modes, becomes negligible (see Fig. 5 and Fig. 7). E.g. when $S/h=1$ the ratio between $Z_{1&6}$ and $Z_{2&5,3&4}$ is $5$–$6$ (see Fig. 5), depending on the permittivity of the dielectric substrate $\varepsilon_r$ (the larger $\varepsilon_r$, the less difference). And when $S/h=10$ the said difference is only $3$–$7\%$ (see Fig. 5). If an MMCL operates in the odd normal mode, the difference between $Z_{1&6}$ and $Z_{2&5,3&4}$ (see Fig. 7), when $S/h=1$, is less than in case of the even normal mode, only $65$–$75\%$. And for $S/h=10$, the said difference is $3$–$7\%$ (see Fig. 7).

Conductors of the synthesized MMCL were divided into $2500$–$4500$ sub-areas (for odd- and even-mode respectively) during all calculations, and the synthesis procedure took $220$–$560$ s (Pentium 4 CPU, $3$ GHz clock frequency and $1$ GB RAM).

Conclusions

Fast and accurate synthesis technique of six-conductor symmetrically coupled MMCL, operating in normal mode, is proposed. In order to demonstrate feasibility of the proposed techniques authors have written specific software and synthesized as well as investigated several MMCLs, operating in the even- and the odd normal mode. It was found, that increasing the space between the adjacent conductors, the MMCL becomes similar to the set of uncoupled microstrips with equal impedance regardless of the operating mode. It was also revealed that external conductors have the most impact on the normal mode in the symmetrically coupled MMCL. Conductors of the synthesized MMCL were divided into $2500$–$4500$ sub-areas (for the odd- and the even-mode respectively) during all calculations and the synthesis procedure took $220$–$560$ s (Pentium 4 CPU, $3$ GHz clock frequency and $1$ GB RAM). The obtained results were compared with the data published by other researchers. Total error was typically less than $3\%$.

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References


Fig. 6. Normalized width of conductors of synthesized six-conductor symmetrically coupled MMCL, operating in odd normal mode, versus normalized space between conductors and permittivity of the dielectric substrate

Fig. 7. Characteristic impedance of synthesized six-conductor symmetrically coupled MMCL, operating in odd normal mode, versus normalized space between conductors and permittivity of the dielectric substrate (external conductors – solid curves, internal conductors – dashed and dotted curves). $W_i/h$ corresponds to Fig. 6


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Synthesis technique of six-conductor symmetrically coupled microstrip multiconductor line (MMCL), operating in a normal mode, is presented in this article. Results of the calculations acquired using the proposed mathematical model of an MMCL differ only by 3% from the results acquired by other researchers. Examples of synthesis of six-conductor symmetrically coupled MMCL, operating in the even- and odd normal modes, are submitted in the article, also, dependences of parameters of these lines are investigated. It was established that the external conductors of an MMCL impart the most for ensuring the normal mode in an MMCL, and the organization of the internal conductors remains regular in this case. It is shown that parameters of the synthesized MMCL operating in the even normal mode depend in greater degree on the space between conductors, than MMCL operating in the odd normal mode. Ill. 7, bibl. 27, tabl. 2 (in English; abstracts in English and Lithuanian).


Sraipsnyje pateikta simetriškai susietų šešių laidininkų mikrojuostelinės linejos (MJDL), veikiančios normaliųjų bangų režimu, sintezė metodika. Skaičiavimų rezultatai, gauti taikant modeliiję sūnų matematinių daugialaidžių linejos modelių, skirsiai nuo kitų tyrimų gautų rezultatų apie 3%. Sraipsnyje taip pat pateikta simetriškai susietos šešių laidininkų MJDL, veikiančios linės ir nelyginius normaliųjų bangų režimų, sintezės pavyzdžių bei išnagrinėtos šių linijų konstrukcijų ir elektrinių parametrų priežiūros. Nustatyta, kad normaliųjų bangų režimu užtikrinti didžiausią įtaką turėjo laidinių MJDL laidininkai, vienų laidinių sandara šiuo atveju išlieka reguliari. Parodyta, kad sintezuojamų MJDL, veikiančių linės normaliosios bangos režimų, parametrai labiau priklauso nuo tarpų tarp laidininkų nei linijų, veikiančių nelyginius normaliosios bangos režimų. Ill. 7, bibl. 27, lent. 2 (anglų kalba; santraukos anglų ir lietuvių k.).