Performance Analysis of Amplify-and-Forward Relay System in Hoyt Fading Channels with Interference at Relay

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Introduction

Diversity at the receiver is a well-known promising avenue for improving mean signal strength and reducing signal level fluctuations in fading channels. Performance improvement using diversity reception is considered in [1, 2, 5, 6, 13–15]. The effect of cooperative diversity on the system performance is analyzed in [3, 4] and [16]. In paper [7] diversity techniques operating over shadowed fading channel was presented, but influence of interference was not analyzed.

Relay communications as a means to improve the range and link reliability has recently rekindled enormous interest in the context of user-cooperative communications. Relay processing can be classified as either amplify-and-forward (AF) or decode-and-forward (DF). There are two types of AF relaying schemes considering two different power constraints at the relay: fixed-gain and channel state information (CSI)-based.

Hoyt distribution is commonly used to describe the short-term signal variation of certain wireless communication systems subject to fading [9] and that distribution is normally observed on satellite links subject to strong ionospheric scintillation. Specifically, the Hoyt channel model has been applied in satellite-based cellular communications to characterize more severe fading conditions than those modelled by Rayleigh [9]. Although considerable attention has been paid to outage probability analysis, few published results for Hoyt fading channels are found in the literature, mainly due to reasons of mathematical tractability. Recently, [10] authors studied the outage probability and the average bit error rate (BER) of the CSI-assisted AF protocol with interference at the relay in Rayleigh fading channel.

Few works that have studied the impact of interference on the AF and DF relaying performance have assumed interference either at the relay(s) or the destination(s). Nevertheless, co-channel interference (CCI) is an important issue. Consideration of CCI is necessary because of the aggressive reuse of frequency channels for high spectrum utilization in cellular systems. It has a very long history for investigating the performances of wireless systems in the presence of CCI. It was shown in [8] that the interference can cause a severe performance degradation. Recently, in [11] authors studied the outage probability and the average bit error rate (BER) of the CSI-assisted AF protocol with interference at the relay in Hoyt fading channel.

In this paper we consider a communication between source $S$ and destination $D$ using relay $R$ where $S$ does not have a direct link to $D$ [11]. All nodes are equipped with a single antenna. The communication in the system is divided into two orthogonal time intervals. In the first time interval, $S$ sends its symbol $s_0$ to $R$ which is supposed to operate in an interference limited environment. Received signal, in the presence of single interference at relay $R$, can be written as

$$y_r = \sqrt{P_i} h_i s_0 + \sqrt{P_j} h_j s_j + n_r,$$  \hspace{1cm} (1)
where $h_{nr}$ is the complex channel between $S$ and $R$ with average fading power $\Omega_{nr}$, $P_r$ is the transmit power, $h_r$ is the channel from interference to $R$ with average fading power $\Omega_{dr}$ independent of $h_{nr}$. $P_d$ is interference average power and $n_r$ is the AWGN at $R$ with variance $\sigma_r^2$. All links are assumed to be subject to Hoyt fading. Transmitted symbols $s_t$ and interfering symbols $s_i$ are assumed to have zero mean and unit variance.

Relay $R$ amplifies signal $y_t$ with gain $G$ which is, in the presence of interference, equal to [8]

$$G = \frac{P_r}{P_r |h_{nr}|^2 + P_d |h_j|^2 + \sigma_r^2}.$$  \hfill (2)

In the second time interval $R$ forwards $y_t$ to $D$. The received signal at $D$ is

$$y_d = h_{dr} G (P_r |h_{nr}|s_t + P_d |h_j|s_j + n_d),$$  \hfill (3)

where $h_{dr}$ is the complex channel between $R$ and $D$ with average fading power $\Omega_{dr}$ independent of $h_{nr}$ and $h_j$, and $n_d$ is the AWGN at $D$ with variance $\sigma_d^2$.

Signal-to-interference plus noise ratio (SINR) of the decision variable can be written as

$$\gamma_{eq} = \frac{P_r |h_{nr}|^2 |h_d|^2}{h_{dr}^2 \left( P_r |h_{nr}|^2 + \sigma_r^2 \right) + \sigma_d^2 |G|^2}.$$

Since $R$ is interference limit (the effect of $n_d$ is negligible), Eq. (4) becomes [11]

$$\gamma_{eq} = \frac{\gamma_1 \gamma_2}{\gamma_{INF} (\gamma_2 + 1) + \gamma_1},$$

where $\gamma_1 = P_r |h_{nr}|^2$, $\gamma_2 = \frac{P_r}{\sigma_r^2} |h_d|^2$, and $\gamma_{INF} = P_d |h_j|^2$.

**Performance analysis**

In this section we analyze important system performance measures such as the outage probability and average BER.

The outage probability, $P_{out}$, is defined as the probability that $\gamma_{eq}$ drops below an acceptable threshold $\gamma_0$

$$P_{out} = \Pr(\gamma_{eq} < \gamma_0) = F_{\gamma_{eq}}(\gamma_0),$$

where $\Pr(\cdot)$ denotes probability and $F_{\gamma_{eq}}(x)$ is the cumulative distribution function (cdf) of $\gamma_{eq}$, which is [11]

$$F_{\gamma_{eq}}(\gamma_{th}) = 1 - \int_0^{\gamma_{th}} \int_0^{\gamma_{INF}} f_{z,w}(z) dzdw,$$

where $f_{z,w}(z)$ and $f_{\gamma_{eq}}(x)$ are probability density functions (pdf) of $\gamma_2$ and $\gamma_{INF}$, respectively. Since $h_{nr}$, $h_{dr}$, and $h_j$ are Hoyt random variables, $\gamma_1$, $\gamma_2$ and $\gamma_{INF}$ are random variables with the following pdfs:

$$f_{\gamma_1}(x) = \frac{(1 + q_1^2) e^{-(1+q_1^2)x}}{2q_1^2 I_0(1-q_1^2)},$$

$$f_{\gamma_2}(x) = \frac{(1 + q_2^2) e^{-(1+q_2^2)x}}{2q_2^2 I_0(1-q_2^2)},$$

$$f_{\gamma_{INF}}(x) = \frac{(1 + q_{INF}^2) e^{-(1+q_{INF}^2)x}}{2q_{INF}^2 I_0(1-q_{INF}^2)},$$

where $\gamma_1 = P_r \Omega_{nr}$, $\gamma_2 = P_r \Omega_j / \sigma_d^2$ and $\gamma_{INF} = P_d \Omega_d$, while $q_1, q_2$ and $q_{INF}$ are fading parameters. The cdf of $\gamma_1$ is

$$F_{\gamma_1}(x) = \int_0^x f_{\gamma_1}(y) dy.$$  \hfill (11)

The cdf $F_{\gamma_1}(x)$ may be written as [10]:

$$F_{\gamma_1}(x) = Q(\alpha(q_1), \frac{x}{\gamma_1}) - Q(\beta(q_1), \frac{x}{\gamma_1}),$$

where $Q(x,y)$ is the Marcum Q function and

$$\alpha(q) = \sqrt{1-q^2} \frac{1+q}{1-q},$$

$$\beta(q) = \sqrt{1-q^2} \frac{1-q}{1+q}.$$  \hfill (13)

After substituting (7)-(13) in (6), the outage probability is

$$P_{out} = 1 - \int_0^{\gamma_{INF}} \int_0^{\gamma_{th}} \left( \frac{\gamma_{th} (w + \gamma_{th} + 1)}{w} \right) \times$$

$$\times \left( \frac{(1 + q_2^2) e^{-(1+q_2^2)x}}{2q_2^2 I_0(1-q_2^2)} \right) \times$$

$$\times \left( \frac{(1 + q_{INF}^2) e^{-(1+q_{INF}^2)x}}{2q_{INF}^2 I_0(1-q_{INF}^2)} \right) dwdz.$$  \hfill (14)

The average BER, derived using 4-QAM modulation format, is equal [12]

$$P_b \approx E\left[ \sqrt{\gamma_{eq}} \right],$$

where $Q(\cdot)$ is the Gaussian Q-function. Because of easier mathematical operations, $\gamma_{eq}$ will be replaced by

$$\gamma_{eq} = \min \left( \frac{\gamma_1}{\gamma_{INF}}, \frac{\gamma_2}{1} \right),$$

as in [8]. Using $\gamma_{eq}$, the average BER may be written as

$$P_b \approx \frac{1}{\sqrt{2\pi}} \int_0^{\gamma_{eq}} f_{\gamma_{eq}}(x^2) \exp(-x^2 / 2) dx,$$  \hfill (17)
where \( F_{\gamma_2} (x) \) is the cdf of \( \gamma_2 \), which is equal to [11]

\[
F_{\gamma_2} (x) = 1 - C_{\infty} (x) C_{\gamma_2} (x),
\]

(18)

where \( C_{\infty} (x) \) and \( C_{\gamma_2} (x) \) are complementary cdfs of \( \gamma_1 / \gamma_{\infty} \) and \( \gamma_2 \), respectively. The pdf of \( \gamma_1 / \gamma_{\infty} \) is

\[
f_{\gamma_1} (x) = \int_{0}^{x} z \cdot f_{\gamma_2} (z) \cdot f_{\gamma_{\infty}} (z) \, dz.
\]

(19)

The complementary cdfs of \( \gamma_1 / \gamma_{\infty} \) and \( \gamma_2 \) are:

\[
C_{\gamma_1} (x) = 1 - \int_{0}^{x} f_{\gamma_1} (x) \, dx,
\]

(20)

\[
C_{\gamma_2} (x) = 1 - \int_{0}^{x} f_{\gamma_2} (x) \, dx.
\]

(21)

After substituting (18), (20), and (21) in (17), by numeric integration we get the average BER.

**Numerical results**

Fig. 1, Fig. 2 and Fig. 3 show results from (14), with assumption \( q_1=q_2=\rho_{\infty}=q \). The strength of the interference is studied using signal-to-interference ratio (SIR) \( \rho = \gamma_1 / \gamma_{\infty} \).

**Fig. 1.** Outage probability as a function of \( q \) for \( \gamma_0 = -5 \text{ dB} \)

Fig. 1 shows the \( P_{\text{out}} \) as a function of \( q \), with \( \rho \) and \( \gamma_0 \) as parameters. It can be seen that \( q \) has stronger influence on the \( P_{\text{out}} \) for higher values of \( \rho \). Also, the \( P_{\text{out}} \) changes more rapidly for smaller values of \( q \). Signal-to-noise ratio (SNR) \( \gamma_0 \) at \( R \) has the same influence on the \( P_{\text{out}} \) for any considered \( q \).

The \( P_{\text{out}} \) as function of \( \gamma_0 \) is shown in Fig. 2. The outage probability threshold \( \gamma_0 \) has stronger impact on the \( P_{\text{out}} \) for lower values of \( \gamma_0 \) and for higher values of \( q \).

**Fig. 2.** Outage probability as a function of \( \gamma_0 \) for \( \gamma_2 = 20 \text{ dB} \)

Fig. 3 shows the \( P_{\text{out}} \) as a function of \( \gamma_2 \), with \( q \) and \( \rho \) as parameters. It can be seen that there is the outage probability threshold for higher values of \( \gamma_2 \) because of the influence of interference.

**Fig. 3.** Outage probability as a function of \( \gamma_2 \) for \( \gamma_0 = -5 \text{ dB} \)

**Fig. 4.** Average BER as a function of \( \gamma_2 \)
The average BER as a function of $\gamma_2$, with $q$ and $\rho$ as parameters, is shown in Fig. 4. Due to impact of interference there is BER floor for higher values of $\gamma_2$. Considering case of no interference, it can be seen that the interference may cause a significant performance loss for higher values of SNR.

Conclusions

Performance of an amplify-and-forward relay system with co-channel interference at relay in Hoyt fading environment are presented in this paper. We consider 4-QAM modulation format. The results show that the outage probability is more influenced by the Hoyt fading parameter $q$ for higher SIR. In the presence of interference there is a BER floor for higher SNR. Acceptable threshold $\gamma_0$ has stronger impact on the outage probability for lower values of $\gamma_0$.

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References


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Performance of an amplify-and-forward relay system, with co-channel interference at relay in Hoyt fading environment is considered. The outage probability and the average bit error rate of the system are determined. The influence of the interference, as well as the influence of other system's parameters on the system’s performance is considered. Ill. 4, bibl. 16 (in English; abstracts in English and Lithuanian).


Pateikiamos stiprinančiųjų ir tiesioginių reišinių perdavimo sistémų su tarpkanalė interferencija Hoito slopinimo kanale charakteristikos. Nustatyta sistemos prastovos tikimybė ir vidutinis bitų klaidų lygis. Nagrinėta interferencijos bei kitų sistemos parametrų įtaka sistemos našumui. Il. 4, bibl. 16 (anguš kalba; santraukos anglų ir lietuvių k.).