Solar Irradiance Model for Solar Electric Panels and Solar Thermal Collectors in Lithuania

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Introduction
As the technology evolves and the price of fossil fuel resources grows rapidly, the increasing focus on renewable energy resources is observed. Much attention is paid to wind power plants [1, 2], systems composed of wind turbines, solar cells and batteries [3].

For the detailed analysis of electronic systems, using solar cells and solar thermal collectors, mathematical models of their constituent components must be developed. The primary input signal for such model is solar energy flux. Since the statistics about solar energy are collected in a limited amount of geographical areas, one must first develop a mathematical model of insolation reaching Lithuanian territory.

The paper describes the developed solar insolation model, based on the clear sky standard and used for electronic systems, which are managing solar cells and solar thermal collectors. Research data on the solar energy flux distribution in Lithuanian territory is presented.

Solar radiation
Solar radiation is a result of a fusion reaction where the hydrogen turns into helium. The amount of solar energy, falling on a unit area depends on latitude, altitude, year and time of a day.

Solar spectrum consists of wavelengths, which fall in the range of visible light, infrared and ultraviolet radiation. Solar radiation is similar to the ideal black body heated to 5800˚ K, radiation [4]. Solar spectrum above the Earth's atmosphere is calculated on the basis of the Planck's law as an absolute black body spectral radiance of electromagnetic radiation at all wavelengths $I(\lambda, T)$

$$I(\lambda, T) = \frac{dW}{dS \, d\lambda} = \frac{8\pi h c}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1},$$ (1)

here $dW$ – energy emitted from area $dS$ during the time $dt$, where the wavelengths are in range from $\lambda$ to $\lambda + d\lambda$; $h$ - Planck's constant, $c$ – speed of light, $k$ – Boltzmann constant, $T$ – absolute temperature of the black body.

Solar insolation $I(T)$ and its spectral radiance $I(\lambda, T)$ related as follows

$$I(T) = \int_{\lambda} I(\lambda, T) \, d\lambda = \frac{8\pi^5 k^4}{15 c^3 h^3} T^4 = \frac{4\pi}{c} T^4.$$ (2)

This relationship is obtained following a Stefan and Boltzmann Law. In expression (2) $\sigma$ denotes the Stefan Boltzmann constant. The coefficient $4\sigma / c$ is called radiation constant.

Extraterrestrial solar insolation
Earth rotates around the Sun in elliptic orbit. Therefore, when calculating extraterrestrial solar energy flux, elliptic orbit is assumed using eccentricity factor $\varepsilon$.

Insolation $I_s(n)$, in the chosen day of the year $n$ is calculated by the following expression

$$I_s(n) = I_0 \varepsilon(n),$$ (3)

here $I_0 = 1367$ W/m² is a solar constant [4].

Sufficiently accurate results are obtained using the eccentricity factor $\varepsilon(n)$ calculated using following expression [4]:

$$\varepsilon(n) = 1 + 0.00011 + 0.03421 \cos \Gamma + 0.00026 \sin \Gamma + 0.00002 \cos 2\Gamma + 0.000079 \sin 2\Gamma,$$ (4)

here $\Gamma$ denotes the day angle in radians

$$\Gamma = \frac{2\pi(n-1)}{365},$$ (5)

here $n = 1, 2, 3, \ldots 365$ is number of the day in a year (January 1 $n = 1$, December 31 is $n = 365$).

According to expressions (3) – (5) it is possible to calculate that approximately on July 5 ($n = 187$) the Earth
is at the furthest point from the Sun – aphelion, and the first days of January and the end of December – closest to the sun – perihelion. For this reason, in the northern hemisphere summers are cooler and winters are warmer.

Fig. 1. Solar radiation falling to the ground plane

Model of the solar energy flux falling on Earth's horizontal surface

On a way to Earth's surface solar rays have to pass Earth’s atmosphere (Fig. 1). Travelling through the atmosphere part of solar rays is absorbed, reflected and scattered. Solar energy flux that is falling directly to the ground plane is estimated using the following expression

\[ I_s(n) = I_s(n) \alpha \cos \theta_Z, \]  

where \( \alpha \) – attenuation of solar energy flux in atmosphere, \( \theta_Z \) – Zenith angle. This angle describes the height of the sun above the horizon. Zenith angle equals the angle formed by the sun's rays falling to land surface and the direction of zenith.

The absorption, reflection and scattering of solar rays depend on the weather and other conditions. Generated model simplification is relied on the clear sky standard definition. As a clear sky the gas medium is assumed, obtained mixing an optically black, gray and transparent gas in equal proportions. Solar flux attenuation at a clear sky atmosphere is calculated as follows [4]

\[ \alpha = a_0 + a_1 \cos h, \]  

where \( a_0, a_1, \) and \( k \) – are the factors characterizing the attenuation of solar radiation in the atmosphere. These factors vary from mist (visibility) in the atmosphere and the observed surface height above sea level. The coefficients \( a_0, a_1, \) and \( k \) for a plane up to 2.5 km above sea level and at 23 km visibility, are calculated according to the following empirical expressions:

\[ a_0 = 0.4237 - 0.00821(6 - h)^2, \]
\[ a_1 = 0.5055 - 0.005958(6.5 - h)^2, \]
\[ k = 0.2711 - 0.01858(2.5 - h)^2, \]

here \( h \) denotes the height above sea level in kilometres.

For calculations of solar energy flux at a horizontal plane, it is convenient to change the Zenith angle \( \theta_Z \) to solar altitude (elevation) angle \( \beta \) (Fig. 1). These angles are linked by the relationship

\[ \cos \theta_Z = \cos \left( \frac{\pi}{2} - \beta \right) = \sin \beta. \]  

Zenith and solar altitude angles depend, as shown in Fig. 2, from observer location A geographic coordinates, distance from the sun to equator (declination) and time of day. Zenith and solar altitude angles used in the model are calculated using following expression [5]

\[ \cos \theta_Z = \sin \beta = \cos \left( \frac{\pi}{180} \phi_{LA} \right) \cos \Delta \phi_M \cos \delta + \sin \phi_{LA} \sin \delta, \]  

here \( \phi_{LA} \) – latitude angle of location A, in deg., \( \Delta \phi_M = \phi_{MN} - \phi_{MA} \) – longitude difference angle between local noon \( \phi_{MN} \) and location A \( \phi_{MA} \), rad, \( \delta \) – distance from the sun to equator (declination) angle, rad.

Fig. 2. Factors influencing zenith angle \( \theta_Z \) at a location A

Declination angle \( \delta \) – is the angle between the Earth equatorial plane and a line joining the Sun and the Earth centres. It defines the axis of the Earth heel, which is a key factor in determining the change of seasons. During the year it is changing, as shown in Fig. 3, from -23.45° (December 21 - Winter Solstice) and 23.45° (June 21 - Summer Solstice). The values of declination angle in radians are calculated using the following simplified expression [5]

\[ \delta = 0.4093 \sin \left( \frac{2\pi(n - 81)}{365} \right). \]  

As the Earth rotates around its axis, an angle \( \Delta \phi_M \) used in the (12) changes by \( 2\pi \) radians during the day. For the practical applications it is useful to relate the model with solar (local) time. This is done by replacing the longitude difference angle between local noon meridian and the location \( \Delta \phi_M \) with the following expression

\[ \Delta \phi_M(t_{ST}) = \omega(12 - t_{ST}), \]  

here \( \omega = \frac{\pi}{12} \) – an angular velocity of the Earth’s rotation around its axis, rad/h, 12 – local noon time, h, \( t_{ST} \) – solar (local) time, \( t_{ST} \in [0,24], \) h. Until noon, \( 0 \leq t_{ST} \)
<12, \( \Delta \phi_M (t_{ST}) \) has positive values, but after the noon, 
12 < \( t_{ST} \leq 24 \), \( \Delta \phi_M (t_{ST}) \) has negative values.

Solar altitude angle cannot be negative. When \( \beta < 0 \)
the sun is behind the horizon and its rays do not fall to the
plane located at A. From these conditions it follows that
zenith angle must always be positive and less than \( \pi/2 \)
, \( 0 < \theta_Z \leq \pi/2 \). For this reason, the condition is introduced
in the model that separates day and night time periods:

\[
I_E (n) = \begin{cases} 
I_S (n) \alpha \cos \theta_Z, & 0 < \theta_Z \leq \pi/2, \\
0, & \text{other } \theta_Z \text{ values.} 
\end{cases} \tag{15}
\]

The results of analysis of solar insolation in Lithuania
territory

Lithuania is located between 20°57' and 26°51' east
longitude meridians and between 53º54' and 56°27' north
latitude parallels. Using the model, developed in
MATLAB environment according to (3) - (15), solar
energy flux rates in 313369 locations of Lithuania were
calculated. The geographic coordinates \( \phi_A, \phi_L \)
and altitudes of analyzed locations there combined in three-
dimensional matrix of 623 x 503. Distances between areas
in meridian direction were taken every 0.01°, and in
parallel direction – every 0.006°. Solar energy flow in the
territory of Lithuania on 21 June ( \( n = 173 \) ) calculation
results are presented in Fig. 4, a.

For calculations it is accepted that \( \Delta \phi_M (t_{ST}) = 0 \) in
expression (12). Therefore values of solar insolation at
each location of Lithuania at noon are presented. The analysis of the Fig. let to show that at noon of June 21
average solar insolation in Lithuania territory is 407 W/m².
Minimal values of insolation of 375 W/m² are observed in
low-lying areas, while maximal
values of 439 W/m², are at highlands (for example on
Juozapinės hill). Thus the deviation of insolation values
from the mean value is ± 8% in territory of Lithuania.

Comparing obtained results with data presented in [6], it
can be stated, that model gives appropriate results for
modelling electronic systems. More detailed conclusions
on the accuracy of the model will be made upon receipt of
data from the solar thermal collector control system [7].

Fig. 4, b shows the solar insolation significantly
changes during the year and the day. Those changes
mostly depend on the change of zenith angle \( \theta_Z \)
and altitude angle \( \beta \). For maximal use of solar energy, planes
of solar panels and solar thermal collectors should be
perpendicular to solar beams. \( \cos \theta_Z \) in expression (12)
can be easily linked with the solar azimuth and tilt angles.

\[
t_{ST} = t_{CT} + \Delta \phi_M (t_{ST}) + v_{ST} (\phi_{MTZ} - \phi_{MA}) + \Delta \phi_L (n), \tag{16}
\]

here \( \Delta \phi_C \) – daylight savings time correction, h. During
the summer, \( i = 1 \), clocks are turned forward by 1 hour.
During the winter, \( i = 0 \), \( \Delta \phi_C = 0 \). \( v_{ST} = 1/15 \) – speed
of solar time change in time zone deg/h, \( \phi_{MTZ} \) – longitude
of meridian for the local time zone, deg west, \( \Delta \phi_L (n) \) –
solar noon time correction due to Earth’s elliptical orbit
around the sun on n–th day, h. The correction is calculated
as follows

\[
\Delta \phi_L (n) = 0.165 \sin \left[ 2 \pi (n-81) \right] - 0.126 \cos \left[ \frac{2 \pi (n-81)}{365} \right] - 0.025 \sin \left[ \frac{2 \pi (n-81)}{365} \right]. \tag{17}
\]

Thus, the model is useful for calculation of optimal
angles of solar panels and solar thermal collectors in any
location of Lithuania [8].
Conclusions

1. The developed model based on the clear sky standard allows calculating solar insolation in each location of Lithuania and is suitable for modelling electronic systems for control of cells and solar thermal collectors.

2. Average insolation in the territory of Lithuania on June 21 is equal to 407 W/m² at noon. Lowest insolation of 375 W/m², is in low-lying areas, while in high areas highest insolation of 439 W/m² is calculated. Thus the deviation of insolation values from the mean value in territory of Lithuania is ± 8%.

3. The changes of solar insolation mostly depend on the change of zenith angle $\theta_0$ and altitude angle $\beta$. For maximal use of solar energy, planes of solar panels and solar thermal collectors should be perpendicular to solar beams. Model is suitable for calculation of optimal angles of solar panels and solar thermal collectors in any location of Lithuania.

References


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