Voltage-Mode CFTA-C Third-Order Elliptic Low-Pass Filter Design and Optimization Using Signal Flow Graph Approach

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Abstract—In this work, two active only grounded-C equivalents of third-order voltage-mode (VM) elliptic low-pass (LP) LC ladder prototype are proposed. As active building blocks (ABBs) the recently introduced current follower transconductance amplifier (CFTA) were used. The first active only grounded-C LP filter employing eight CFTAs was proposed by interconnecting CFTA-based active equivalent sub-blocks of passive components, where one of the low-impedance input terminals is not used. Since such feature may cause some noise injection into the proposed circuit, the proposed filter was optimized using Mason-Coates’ signal flow graph approach. In several steps the number of ABBs was reduced by two and the unused input terminal was eliminated. The performance of the novel and optimized active only grounded-C third-order VM elliptic LP filter was tested experimentally using the readily available UCC-NIB integrated circuit.

Index Terms—Active equivalent, analog filters, current follower transconductance amplifier, CFTA, experiments, Mason-Coates signal flow graph, LC ladder filter, optimization, passive prototype.

I. INTRODUCTION

Analog filters have wide area of applications in instrumentation, automatic control, and communication systems. It is well-known that filters with good frequency selectivity have to be of the order higher than two. During the last decades it was shown that LC ladder structures have minimum sensitivity to component variations in the frequency band of interest. Thus, the performance of these types of passive filter structures is very reliable and stable [1]. Elliptic or so-called Cauer filters represent a specific type of LC ladder filters having the transmission zeros as well as poles at finite frequencies that create equal-ripple variations in both the pass-band and the stop-band and feature faster transition from the pass-band to the stop-band than any other class of network synthesis filters [2]. However, on-chip spiral inductors occupy large chip area and therefore are costly and suffer from substrate resistive losses and capacitive couplings. Moreover, their value in passive form is not easily tunable [3]. Due to these disadvantages, after introducing active filters, it has become a common practice to reproduce the operation of ladder passive filters by means of active filter counterparts to maintain the same low-sensitivity characteristics. One of the most powerful methods for synthesis of LC ladder filters is the linear transformation (LT) technique. The principle is based on the linear transformation of port variables of a network from the V-I domain to a new domain, in which active realizations are effected [4], [5]. In other words, LT active filters realize systematic design tables i.e. every section of the original ladder prototype is realized by using active building blocks (ABBs) individually. In general, in the open literature various third-order low-pass filter (LPF) realizations exist [6]–[13], however this paper is strictly focused on active only grounded-C third-order voltage-mode (VM) elliptic LPF design [10]–[13]. Therefore, for fair comparison of here presented solution the operational transconductance amplifier (OTA) [10] and second-generation current conveyors (CCIs)-based solutions are relevant [11], [12]. Both OTA and CCI are suitable for LT filter synthesis, because they have high-impedance input. In [10], the active only grounded-C realization employs seven OTAs, while in [11] and [12] six CCIs, three voltage followers, six resistors (including floating ones), and five CCIs and six grounded resistors are used, respectively. In [13], the signal flow graph (SFG) approach [14] was used for third-order elliptic LPF design. Here, the original ladder network is divided into subsections and then using SFG each subsection is realized one by one. Hence the low sensitivity basis is guaranteed while reducing the complexity of a large signal flow graph diagrams. On the other hand, although both methods are attractive for third-order elliptic LPF design, none of these two methods consider filter structure

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optimization. Therefore, the main aim of this paper is to combine both LT technique and SFG approach such that firstly the active only grounded-C equivalent of third-order VM elliptic LPF is simply designed by LT, i.e. by replacement of passive components by theirs active equivalents and the SFG approach is with advantage used as powerful tool for the LT structure optimization.

The paper is organized as follows: Section II describes the recently introduced ABB so-called current follower transconductance amplifier (CFTA), which is in this paper used for filter design. Part of this section also shortly deals with the Mason-Coates’ SFGs definition and evaluation. In section II-C the CFTA-based new active only grounded-C third-order VM elliptic low-pass filter and its optimized circuit solution are presented. Section III discusses experimental verification, while IV concludes the paper.

II. CIRCUIT DESCRIPTION

A. Description of CFTA+/-

In several earlier reported circuits the potential of the CFTA was not fully used, since one of the input terminals p or n of input sub-block current differenting unit (CDU) is not connected into the proposed function block [15] [21]. Thus, this may cause some noise injection into the IC circuit. Hence, in order to prevent this potential drawback of future applications, the CFTA was introduced as novel ABB for analog signal processing in 2008 [22] [26]. The CFTA+/+ is a four-terminal ABB, which circuit symbol and behavioural model are shown in Fig. 1. Basically, it consists of an input positive current follower stage, which transfers the input current to the z terminal and an OTA stage that converts the voltage at the z terminal to output currents at the x+ and x- terminals. Using standard notation, the relationship between port currents and voltages of non-ideal CFTA+/+ can be described by the following hybrid matrix:

\[
\begin{bmatrix}
  i_+ \\
  i_- \\
  i_f \\
\end{bmatrix} =
\begin{bmatrix}
  Y_z & 0 & 0 & \alpha(s) \\
  g_m(s) & Y_{x+} & 0 & 0 \\
  -g_m(s) & 0 & Y_{x-} & 0 \\
\end{bmatrix}
\begin{bmatrix}
  v_z \\
  v_{x+} \\
  v_{x-} \\
\end{bmatrix},
\]

where \( g_m(s) \) and \( \alpha(s) \) represent frequency dependent transconductance gain from the z terminal to x+ and x terminals and frequency dependent non-ideal current gain, respectively. The \( Z_t = R_t \) is parasitic intrinsic input resistance and \( Y_k = sC_k + 1/R_k \) for \( k \in \{ z, x+, x- \} \) are parasitic admittances at corresponding terminals of CFTA+/+, respectively. Note that in ideal case the current gain is unity, i.e. \( \alpha(s) = 1 \), and frequency independent.

Using a single-pole model it can be defined as \( a(s) = a_0/(1 + \tau_a s) \), where \( a_0 \) is dc current gain, \( 1/\tau_a \) is bandwidth dependent on the IC fabrication of ABB, however, in current CMOS or BiCMOS technologies the bandwidth is in order of a few Grad/s. Hence, at low and medium frequencies, i.e. \( f = (1/2\pi) \times \min\{1/\tau_a\} \), the frequency dependent current gain \( a(s) \) turns to \( a(s) \cong a_0 = 1 + \varepsilon_a \), whereas \( \varepsilon_a \) is current tracking error and satisfies \( |\varepsilon_a| < 1 \). It should be also mentioned that depending on specific implementation of the CFTA+/+ its above mentioned parasitic intrinsic input resistance and non-ideal current gain can be with advantage used as current-controlled tunable parameters. In such cases the current-controlled and/or controlled-gain CFTA could be elaborated.

B. Mason-Coates’ Signal Flow Graphs

For the design and optimization of the active frequency filters based on the passive prototype, the SFG approach has been used. To be able to follow the design and optimization steps the following paragraph shortly describes the evaluation of the transfer function of an M-C (Mason-Coates) SFG.

It is known that the transfer function of an M-C SFG can be determined using the equation also labelled as Mason’s gain formula [14]

\[
K = \frac{Y}{X} = \frac{1}{\Delta} \sum_i P_i \Delta_i, \tag{2}
\]

where \( P_i \) is the transfer of the \( i \)-th direct path from the input current or voltage node \( X \) to the output current and voltage node \( Y \), and \( \Delta \) is the determinant of a graph that is given as follows

\[
\Delta = V - \sum_k S_{1(k)} V_{1(k)} + \sum_l S_{2(l)} V_{2(l)} - \sum_m S_{3(m)} V_{3(m)} + \ldots \tag{3}
\]

where \( V \) is the product of the self-loops, \( S_{1(k)} \) is the transfer of the \( k \)-th oriented loop, and \( V_{1(k)} \) is the product of all self-loops not-touching the \( k \)-th oriented loop, \( S_{2(l)} \) is the transfer product of two not-touching oriented loops, and \( V_{2(l)} \) is the product of the self-loops not-touching the \( l \)-th oriented loops. If an oriented loop or \( k \)-th direct path is touching all nodes, then the product \( V \) or \( \Delta \) is unity. In (2), \( \Delta \) is the determinant of that part of the graph that is not touching the \( i \)-th direct path.

Except the knowledge of evaluating the transfer function of an M-C graph, using the flow graph theory for synthesis of circuits, also the corresponding M-C graph of the active element must be known. According to (1) the corresponding M-C graph of an ideal CFTA+/+ active element is shown in Fig. 2.
Replacing resistors $R_2$ is shown in Fig. 3(a) and can be used to floating capacitor $C_1$. Frequency Filter Prototype (b) is third order VM elliptic low-pass, can be done NO CFTA [26], the CFTA-C frequency filter realization is obtained as shown in Fig. 4(a). The corresponding M-C flow graph of this solution is shown in Fig. 4(b) and can be used to evaluate the voltage transfer function, which has a form

$$K_{act} = \frac{s^2 L C_1 G_1}{s^3 a_1 + s^2 a_2 + s a_3 + a_4}, \quad (4)$$

where $a_0 = G_1 + G_2$, $a_1 = C_1 + C_2 + L_4 G_1 G_2$, $a_2 = L_4 [C_1 (G_1 + G_2) + L_4 C_1 G_2]$, and $a_3 = L_4 [C_1 C_2 + C_3 (C_1 + C_2)]$.

The equivalent M-C graph of the passive LC ladder filter from Fig. 3(a) is shown in Fig. 3(b). Replacing resistors $R_1$ and $R_2$, floating capacitor $C_1$ and inductor $L_1$ in the passive prototype (Fig. 3(a)) by their corresponding representations employing only CFTAs as active elements and/or capacitors [26], the CFTA-C frequency filter realization is obtained as shown in Fig. 4(a). The corresponding M-C flow graph of this solution is shown in Fig. 4(b) and can be used to evaluate the voltage transfer function, which has a form

$$K_{act} = \frac{s^2 C L_1 C_1 G_1 m_1 G_2 m_2 + G_2 m_3 G_3 m_4 + G_3 m_5 G_4 m_6 + G_4 m_7 G_5 m_8}{s^3 b_1 + s^2 b_2 + s b_3 + b_4} \quad (5)$$

where $b_1 = C_{L1} (C_1 C_2 G_3 m_5 + C_2 C_3 G_4 m_6 + C_1 C_3 G_5 m_7)$, $b_2 = C_{L1} [C_1 (G_2 m_3 G_4 m_5 + G_2 m_4 G_3 m_6) + G_2 m_5 G_3 m_7 + G_2 m_6 G_4 m_8]$, and $b_3 = G_3 m_7 (G_3 m_5 G_4 m_6 + G_2 m_4 G_3 m_7)$.

Comparing the transfer functions (4) and (5), the active only grounded-C third-order VM elliptic LPF from Fig. 4(a) generally provides the third-order elliptic low-pass response as required. However, as it can be evident, by simple interconnection of corresponding active replacements of replaced passive elements, the proposed filter is quite excessive in the required number of active elements point of view. Therefore, optimization steps can be done that lead to reducing the number of active elements in the final active CFTA-C third-order VM elliptic low-pass filter solution.
To follow the optimization steps, the numbering of main active elements remains the same. Using an additional current output x in 3MO-CFTA (multi-output CFTA), the 3CFTA+/ in the original solution given in Fig. 4(a) can be omitted in a very simple way as it can be seen from Fig. 5(a) showing the optimized circuit solution. Similarly, the 3CFTA+/ and 3CFTA+/ in the original solution can be joined, where in the optimized circuit only the 3MO-CFTA is presented featuring multiple current outputs x+ and x−. In order to prevent potential noise injection into the on-chip circuit or fabricated prototype, the final optimization step consists in employing the f terminal of 3CFTA+. To ensure that the transfer function does not change, x2, current output of 3MO-CFTA must be used.

The transfer function of the optimized active only grounded-C third-order VM elliptic low-pass filter can be expressed as

$$K_{act, opt} = \frac{s^2C_{L1}C_{C1}g_{m1}g_{m2} + g_{m1}g_{m2}g_{m4}g_{m7}}{s^3c_3 + s^2c_2 + sc_1 + c_0}, \quad (6)$$

where

$$c_3 = C_{L1}(C_{C1}g_{m2} + C_2g_{m7} + C_2C_{C1}g_{m5}),$$
$$c_2 = C_{L1}(2C_{C1}g_{m2}g_{m5} + g_{m7}(C_3g_{m2} + C_2g_{m5})), $$
$$c_1 = g_{m7}(C_2g_{m4}g_{m5} + C_3g_{m2}g_{m4} + C_1L_1g_{m2}g_{m5}),$$
$$c_0 = 2g_{m2}g_{m4}g_{m5}g_{m7}.$$

Comparing the transfer functions (6) and (4), the optimized structure of active CFTA-C filter from Fig. 5(b) also provides the third-order elliptic low-pass response as required and hence the optimization steps are correct. In addition, after the optimization steps, the number of active elements compared with the first CFTA-C filter realization shown Fig. 4(a) was reduced by two, which may reduce its chip area in case of on-chip fabrication.

III. MEASUREMENT RESULTS

In order to confirm the theoretical study and to show the performance of the optimized active only grounded-C third-order VM elliptic LPF from Fig. 5(b), its behaviour has been verified by experimental measurements. To implement the CFTA+/ and MO-CFTAs, the readily available UCC-N1B integrated circuit (IC) developed in the CMOS 0.35 µm technology, which implements the universal current conveyor (UCC) and second-generation current conveyor CCII+–, has been used [27], [28]. The realization of the MO-CFTA by means of UCC-N1B is shown in Fig. 6a, where the grounded resistor R_K defines the transconductance of the active element, whereas g_m = 1/R_K.

The third-order elliptic low-pass filter [29] was designed with the following specification: cut-off frequency 110 kHz, stopband frequency 205 kHz, passband ripple 1 dB, and minimum stopband attenuation 30 dB. The passive element values in the optimized version of the proposed active only CFTA-C third-order VM elliptic LPF from Fig. 5(b) have been determined as follows: C_{C1} = 3.9 nF, C_{L2} = 3.9 nF, C_{L3} = 27 nF, and g_{m0} = 1/R_K = 1/100 Ω for i ∈ {1, 2, 4, 5, 6, 7}. The developed PCB (printed circuit board) is shown in Fig. 7 and the experimental measurements have been carried out using network-spectrum analyser Agilent 4395A.
Both ideal and measured gain responses are shown and compared in Fig. 8. In addition, the screenshot from the network-spectrum analyser showing measured gain and phase responses are given in Fig. 9. The value of the cut-off frequency determined from measurements is approx. 98 kHz. The decrease in the cut-off frequency is caused by the parameters of the used UCC-N1B ICs [27], [28], however, the real behaviour of the filter is still very satisfactory and experimental results confirm the theoretical study.

IV. CONCLUSIONS

In this paper, the Mason-Coates’ signal flow graph approach is demonstrated as powerful tool for third-order voltage-mode elliptic LPF optimization. First, the number of active elements was reduced by two. Second, the not connected low-impedance input terminal was eliminated preventing potential noise injection into the fabricated PCB during experiments. The designed CFTA-based final solution is the first third-order VM elliptic low-pass filter in the open literature. From the experimental results it can be observed that the cut-off frequency precisely agrees to theoretically predicted one. Note that the in higher frequency region the filter characteristics are partly affected by the real properties of the used ICs. However, since the attenuation in full frequency range is below 30 dB, the results are really favourable.

REFERENCES
