Introduction

Phasor diagrams and equivalent circuits are widely used for analysis of all AC electrical machines. Different variants of the phasor diagram for the salient pole synchronous machine are composed using the principle of two reactions, which was proposed by A. Blondel in 1895. Variants of the equivalent circuit of such machines are known as well. Despite the fact that the phasor diagrams and equivalent circuits for the considered machines have been used for a long time, development of those by various evaluation of the machine power is presented in this paper.

Main assumptions

The phasor diagrams and equivalent circuits are examined neglecting the armature winding resistance (because evaluation of this resistance does not change principal results). The unsaturated machine is examined. Phasor diagrams are presented for generator action, showing voltage drops across the corresponding reactances (and resistances).

Phasor diagram for the non-salient pole synchronous machine

First of all the phasor diagram for the non-salient pole synchronous generator could be reminded (Fig. 1a).

Calculation of the output power according to the phasors \( \mathbf{U} \) and \( \mathbf{I} \) yields well-known expression:

\[
P = \frac{mE}{x_s}\sin \theta,
\]

where \( m \) is the number of phases, \( x_s \) is the synchronous reactance.

The same expression can be got according to phasors \( \mathbf{E} \) and \( \mathbf{I} \), that is according to developed power

\[
P = mE\cos \psi = mE\frac{x_sI\cos \psi}{x_s} = \frac{mEU}{x_s}\sin \theta.
\]

Phasor diagram for the salient pole synchronous machine

Phasor diagram for the salient pole synchronous generator can be constructed from the following equation (that is, finding the terminal voltage \( \mathbf{U} \) by subtracting voltage drops from the excitation electromotive force \( \mathbf{E} \)):

\[
\mathbf{U} = \mathbf{E} - jx_d \mathbf{L}_d - jx_q \mathbf{L}_q,
\]

where \( \mathbf{L}_d \) is the direct-axis current, \( \mathbf{L}_q \) is the quadrature-axis current (the corresponding components of the armature current \( \mathbf{I}_d, \mathbf{I}_q \) is the direct-axis synchronous reactance, \( x_q \) is the quadrature-axis synchronous reactance. It is to notice that these reactances are treated as fixed parameters.

The conventional phasor diagram according to (3) is shown in Fig. 2.

Calculation of the output power from the phasors \( \mathbf{U} \) and \( \mathbf{I} \) results well-known expression:

\[
P = \frac{mEU}{x_d}\sin \theta + \frac{mU^2}{2} \left( \frac{1}{x_q} - \frac{1}{x_d} \right)\sin 2\theta.
\]
Still another expression for power results when the developed power is calculated according to the phasors $E$ and $I_q$ (because the phasors $E$ and $I_d$ are perpendicular):

$$P = mEI_q = \frac{mEU}{x_q} \sin \theta. \quad (5)$$

This wrong result is got despite the fact that phasors of the voltage drops across the corresponding synchronous reactances and phasors of the current components are always in quadrature, and these pairs of phasors develop any power. That signifies that the conventional phasor diagram, as is shown in Fig. 2, is not complete. Possible complement of such diagram is presented in the next section.

Exact expression of the machine power should be derived from the conventional phasor diagram (Fig. 2), if the sum of voltage drops across the synchronous reactances $x_q$ and $x_d$ was evaluated as the voltage drop across some equivalent non-linear reactance $x_{sn}$. Such phasor diagram is presented in Fig. 3.

It is easy to ascertain that the sum of (5) and (6) leads to exact expression of the salient pole synchronous machine power (4):

$$\frac{mEU}{x_q} \sin \theta + \frac{mEU}{x_d} \left( \frac{1}{x_d} - \frac{1}{x_q} \right) \sin \theta \cdot \sin \theta =$$

$$= \frac{mEU}{x_d} \sin \theta + \frac{mU^2}{2} \left( \frac{1}{x_q} - \frac{1}{x_d} \right) \sin 2\theta. \quad (7)$$

As we can see, the presented in Fig. 3 development of the phasor diagram enables to calculate rightly the machine power in different way.

**Equivalent circuit for the salient pole synchronous machine**

Construction of equivalent circuit for the salient pole synchronous machine based on the voltage equation (3) are problematical because it is necessary to sum up the voltage drops caused by different components of the armature current. Therefore mostly only abstract view of an equivalent circuit is presented, as it is shown in the sample of Fig. 4 [1].

As we can see in Fig. 4, the synchronous reactance is depicted in generalised mode (as some set of the direct-axis and quadrature-axis synchronous reactances). Here the components of armature current are presented schematically too.

There are other variants of the considered equivalent circuit. For example, we can find an equivalent circuit, which is composed according to principle of the transformer equivalent circuit (e.g. Fig. 5 [2]). Unfortunately, such equivalent circuit is not convenient to analyse salient pole synchronous machine.

Naturally, the generalised equivalent circuit can be presented with non-linear synchronous reactance $x_{sn}$, as it...
is shown in Fig. 6. However, such equivalent circuit also is not convenient to analyse the considered machine.

![Fig. 6. Generalised equivalent circuit for the salient pole synchronous machine](image)

If we would like to specify the equivalent circuit, first of all the armature current $I$ must bifurcate into two branches with different components of the current $I_d$ and $I_q$. This procedure is presented in Fig. 7.

![Fig. 7. Specified equivalent circuit for the salient pole synchronous machine](image)

The current $I$ is divided in the branch points into their components shunting the direct-axis and quadrature-axis reactances by additional elements $z_q$ and $z_d$ (indefinite thus far). Parameters of these elements are determined by condition that voltage of the parallel elements caused by different components of the current are the same. Thus, this condition gives:

$$jx_d L_d = z_q L_q,$$  \hspace{1cm} (8)
$$jx_q L_q = z_d L_d.$$  \hspace{1cm} (9)

Upon solving equations (8) and (9) for $z_q$ and $z_d$, there results:

$$z_q = r_q = x_d \tan \psi,$$  \hspace{1cm} (10)
$$z_d = r_d = -x_q \cot \psi.$$  \hspace{1cm} (11)

It is naturally that the addition elements are the resistances because of the phasors $L_d$ and $L_q$ are being in quadrature. On the other hand, these additional resistances are depending on the phase angle $\psi$, that is, are non-linear. It is very important that the resistance $r_d$ is negative (that is possible in equivalent circuits).

The equivalent circuit in Fig. 7 can be rearranged into ultimate form, which are shown in Fig. 8.

![Fig. 8. Developed equivalent circuit for the salient pole synchronous machine](image)

It is to accentuate that the non-linear reactance $x_{sn}$ (Fig. 6) in fact has included some resistances $r_q$ and $r_d$ (positive and negative).

The developed electrical power of the considered machine can also be expressed from the equivalent circuit depicted in Fig. 8. In this case, it is necessary to subtract from the power according to phasors $\mathbf{E}$ and $\mathbf{I}$ (5) the power $\Delta P_r$ in resistances of the equivalent circuit $r_d$ and $r_q$. Thus, this power $\Delta P_r$ (per phase) is:

$$\Delta P_r = r_d I_d^2 + r_q I_q^2 = -x_q \cot \psi \cdot I_d^2 + x_d \tan \psi \cdot I_q^2 =$$
$$= -x_q \frac{I_d}{I_d} I_d^2 + x_d \frac{I_d}{I_q} I_q^2 = I_d I_q (x_d - x_q) = -\Delta P.$$  \hspace{1cm} (12)

As we can see, this power is equal to the negative additional power from the intermediate expression (6) (in fact, this is an additional power developed in the salient pole synchronous machine due to variable permeance [3]). Consequently, the calculated according to the equivalent circuit Fig. 8 electrical power will be the same true power presented in expression (4). This situation corroborates expedience of the proposed equivalent circuit for the salient pole synchronous machine.

The phasor diagram can also be constructed from the equivalent circuit. Such phasor diagram is shown in Fig. 9. In fact, here the completed conventional (see Fig. 2) phasor diagram is got. Naturally, the true electrical power of the considered machine can be calculated in any way from this phasor diagram. So, the proposed equivalent circuit enables to complement properly the conventional phasor diagram for the salient pole synchronous machine.

![Fig. 9. Complemented phasor diagram for the salient pole synchronous machine](image)

Conclusions

Traditional phasor diagram for the salient pole synchronous machine constructed on a basis of two reactions is not complete, and it enables to calculate machine power only in separate way (from the $\mathbf{U}$ and $\mathbf{I}$ phasors).

It is possible to develop an equivalent circuit for the salient pole synchronous machine, in which the direct-axis and quadrature-axis synchronous reactances and additional resistances are presented, as well as the corresponding components of armature current are shown.

The developed phasor diagrams and equivalent circuits for the salient pole synchronous machine enable to calculate power of the machine in various ways.
References


Phasor diagrams and equivalent circuits are widely used for analysis of AC electrical machines for a long time. Different variants of the phasor diagram for the salient pole synchronous machine are composed using the principle of two reactions. Variants of the equivalent circuit of such machines are known too. Traditional phasor diagram enables to calculate machine power only in separate way (from the phasors of external voltage and armature current). The developed phasor diagrams and equivalent circuit, in which the direct-axis and quadrature-axis synchronous reactances and additional resistances with the components of armature current are used, are presented in the paper. They enable to calculate power of the considered machine in various ways and more successful to analyse them.


Векторные диаграммы и схемы замещения уже долгие годы используются для анализа электрических машин переменного тока. Различные варианты векторных диаграмм синхронных явнополюсных машин строятся на основе метода двух реакций. Однако традиционные векторные диаграммы позволяют выразить мощность машины единственным способом – с использованием векторов внешнего напряжения и тока якоря. В статье представлены развитые векторные диаграммы, а также схема замещения с использованием продольного и поперечного синхронных реактивных сопротивлений и добавочных активных сопротивлений с выделением составляющих тока в отдельных ветвях. Это позволяет получить выражение мощности машины различными способами, а также более удобно анализировать свойства машины. Ил. 9, библ. 3 (на английском языке; рефераты на английском, русском и литовском яз.).


Кинематические диаграммы и схемы замещения уже долгие годы используются для анализа электрических машин переменного тока. Векторные диаграммы и схемы замещения позволяют выразить мощность машины единственным способом – с использованием векторов внешнего напряжения и тока якоря. В статье представлены развитые векторные диаграммы, а также схема замещения с использованием продольного и поперечного синхронных реактивных сопротивлений и добавочных активных сопротивлений с выделением составляющих тока в отдельных ветвях. Это позволяет получить выражение мощности машины различными способами, а также более удобно анализировать свойства машины. Ил. 9, библ. 3 (англ. kalba; santraukos anglų, rusų ir lietuvių k.).